

# Strong gravitational lensing in cosmology

**Aleksandra Piórkowska**

Institute of Physics, University of Silesia, Katowice, Poland



# Observational cosmology group (University of Silesia)

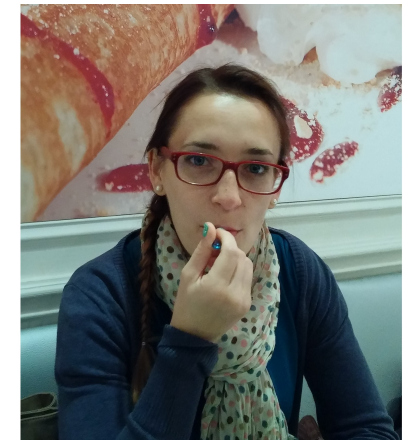
Leader of the group:  
professor M. Biesiada



dr A. Piórkowska



dr B. Malec



mgr S. Miernik  
(PhD student)

- Area of interest:

- dark energy problem in the Universe
- astrophysical limits on exotic theories (including dark matter)
- astrophysical and cosmological gravitational wave sources

# Outline

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- Introduction – gravitational lensing in (real) short
- Strong gravitational lensing as a tool in cosmology:
  - dark energy problem (very briefly)
  - distance measures
  - strong lenses as standardizable rulers – our method and latest results
  - discussion
- Summary and prospects

# Gravitational lensing in a pill

- Deflection of the light path near massive body can be calculated within Newtonian theory of gravity

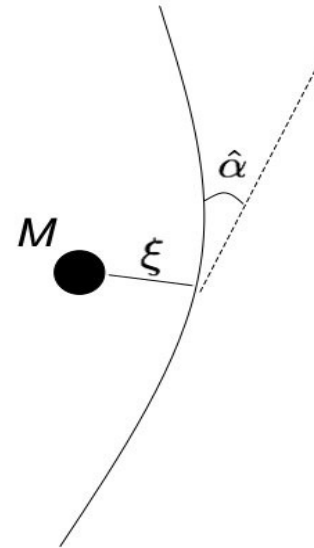
test particle with velocity  $v$  moving past an object of mass  $M$  is deflected by

$$\hat{\alpha} = 2GM/(v^2\xi)$$

if light treated as particles

$$\hat{\alpha}_N = 2GM/(c^2\xi)$$

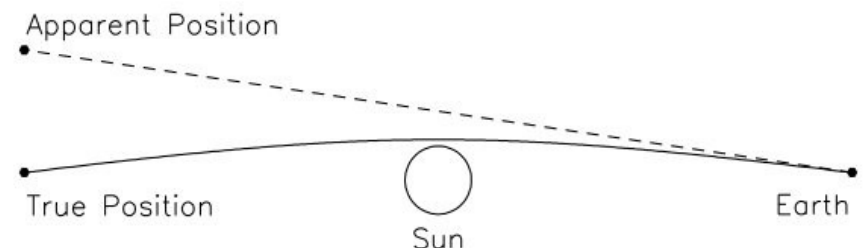
[Mitchell 1784; Soldner 1804]



after S. Suyu; lectures XXIV Canary Islands Winter School of Astrophysics 2012

- But the deflection angle derived from general theory of gravity is as twice as it:

$$\hat{\alpha}_E = 4GM/(c^2\xi) = 2\hat{\alpha}_N$$

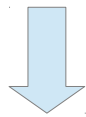


A. Einstein (1915); proved by A. Eddington in 1919

# Gravitational lensing in a pill

- The effect of spacetime curvature on the light paths can be expressed in terms of an effective index of refraction  $n$ :

$$n = 1 - \frac{2}{c^2} \Phi = 1 + \frac{2}{c^2} |\Phi|$$

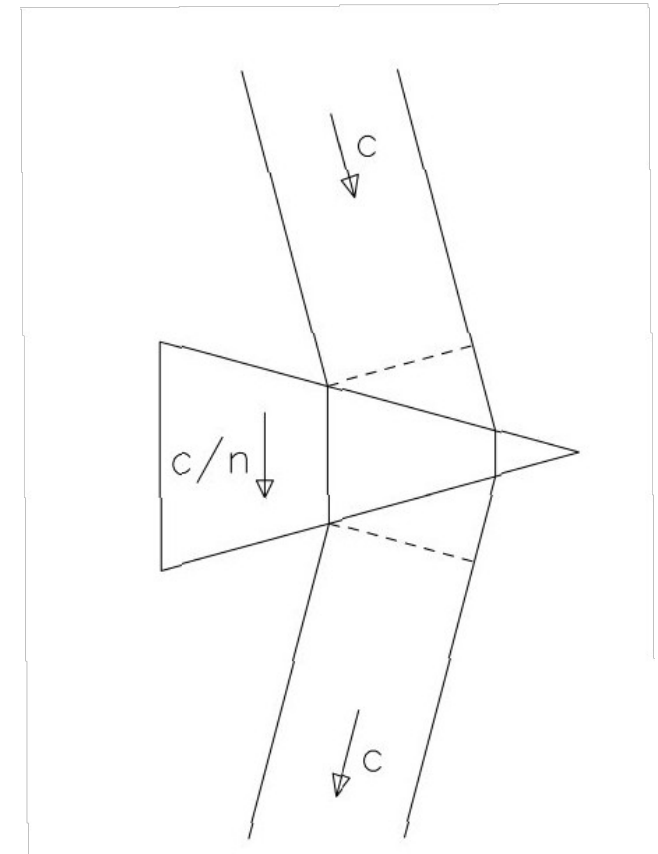


$$v = \frac{c}{n} \simeq c - \frac{2}{c} |\Phi|$$

- Deflection is the integral along the light path of the gradient of  $n$  perpendicular to the light path:

$$\vec{\alpha} = - \int \vec{\nabla}_{\perp} n dl = \frac{2}{c^2} \int \vec{\nabla}_{\perp} \Phi dl$$

Schneider et al.1992



After M. Bartelmann 1996

# Gravitational lensing in a pill

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- The Newtonian potential of the point mass lens:

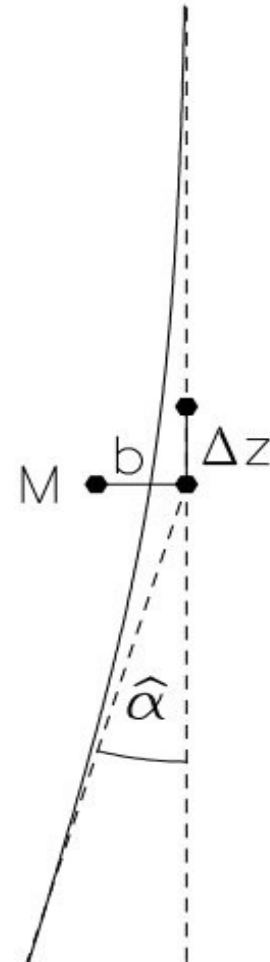
$$\Phi(b, z) = -\frac{GM}{(b^2 + z^2)^{1/2}}$$

- Deflection angle in this case is:

$$\hat{\alpha} = \frac{2}{c^2} \int \vec{\nabla}_{\perp} \Phi dz = \frac{4GM}{c^2 b}$$

**twice the inverse of the impact parameter  
in units of the Schwarzschild radius**

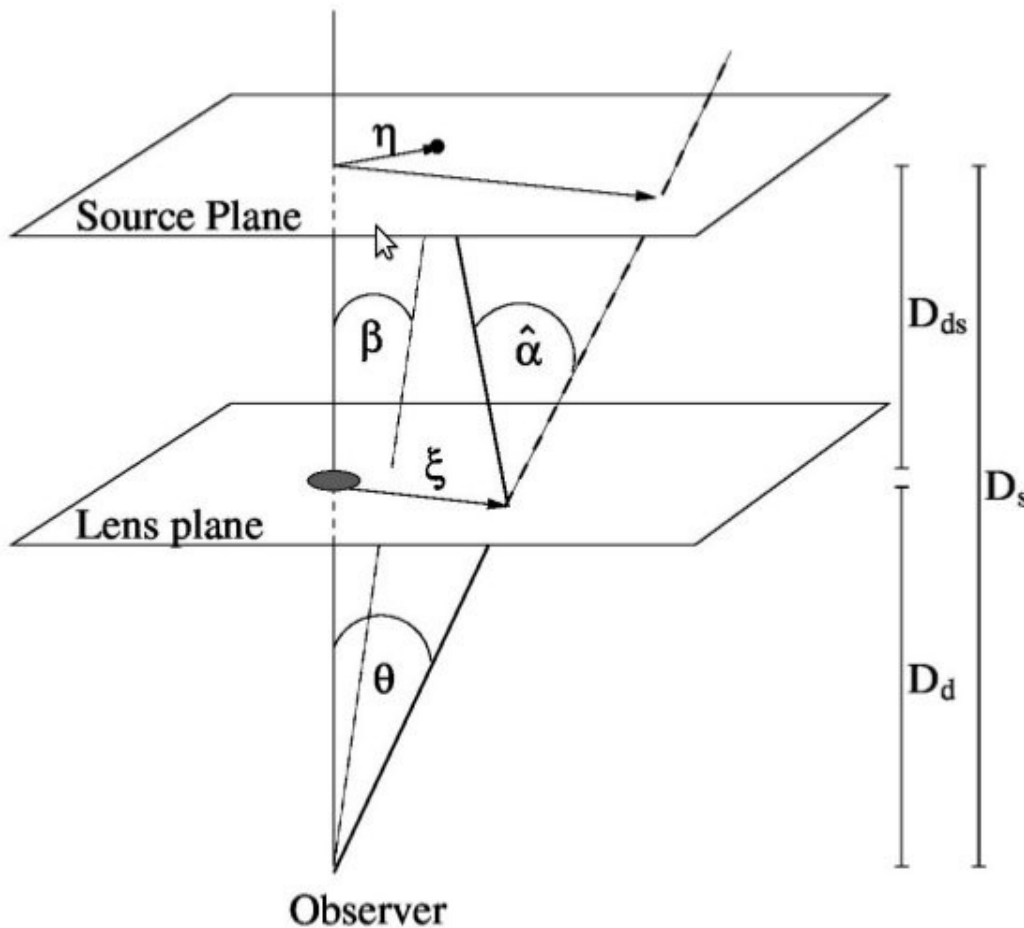
for the light ray deflected by  
the Sun the angle is 1."7



After M. Bartelmann 1996

# Gravitational lensing in a pill

- Geometry of a typical gravitational lens system gives the so-called Lens (ray-tracing) equation:



$$\eta = \frac{D_s}{D_d} \xi - D_{ds} \hat{\alpha}(\xi)$$

In terms of angular coord.:

$$\eta = D_s \beta$$

$$\xi = D_d \theta$$

$$\beta = \theta - \alpha(\theta)$$

where

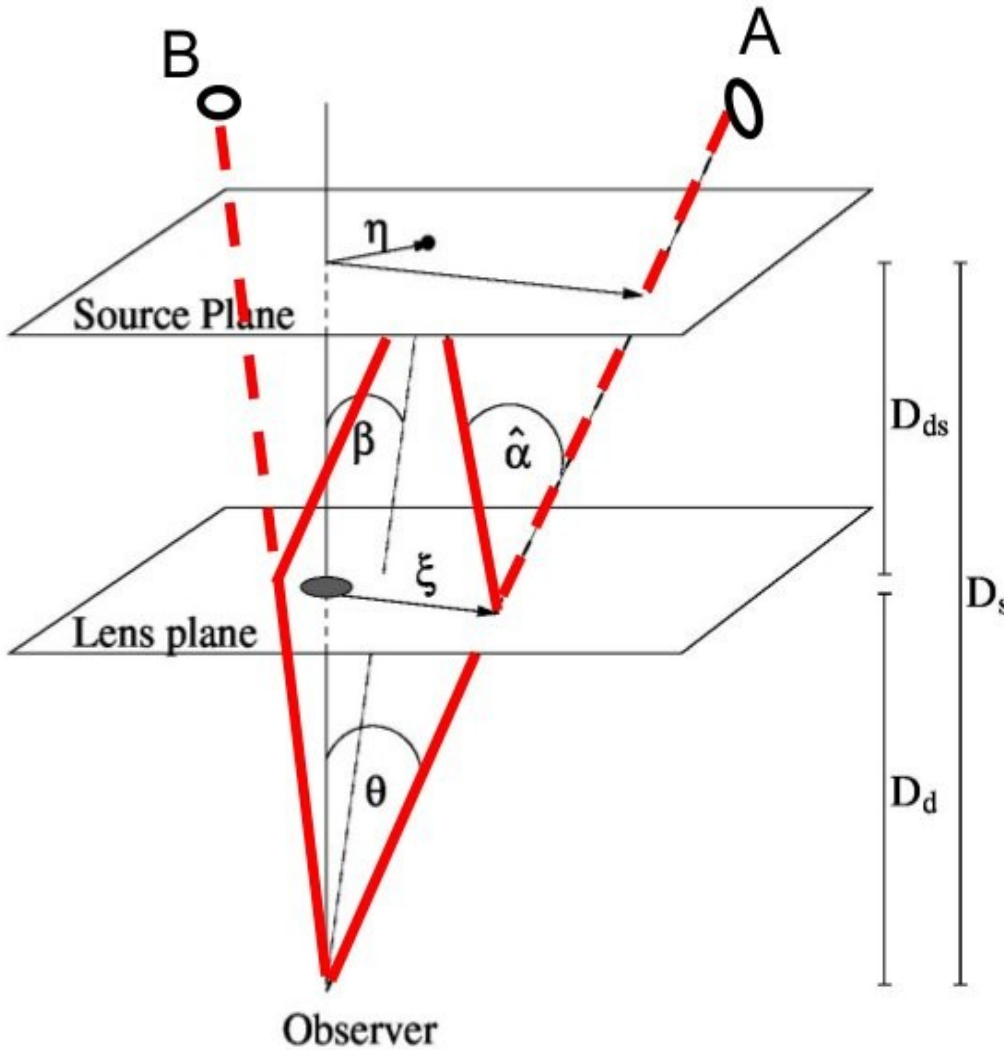
$$\alpha(\theta) = \frac{D_{ds}}{D_s} \hat{\alpha}(D_d \theta)$$

reduced deflection angle

[Schneider et al. 2006]

# Gravitational lensing in a pill

- Gravitational lensing time delay:



differences between travel times for light rays from different images

$$t(\vec{\theta}) = t_{\text{geom}} + t_{\text{grav}}$$

Shapiro time delay

$$t_{\text{grav}} = \int_{\text{source}}^{\text{observer}} \frac{2}{c^3} |\Phi| dl$$

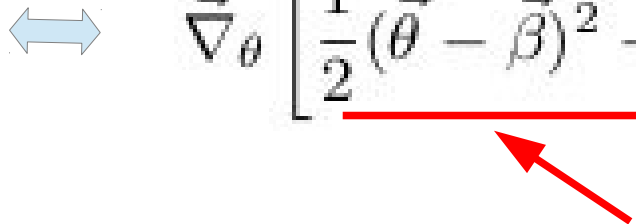
[Schneider et al. 2006]



# Gravitational lensing in a pill

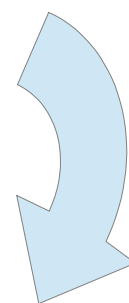
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- Gravitational lensing and Fermat principle:

$$(\vec{\theta} - \vec{\beta}) - \vec{\nabla}_{\theta} \psi = 0 \quad \Leftrightarrow \quad \vec{\nabla}_{\theta} \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi \right] = 0$$


Time delay function:

**The so-called Fermat potential**

$$t(\vec{\theta}) = \frac{(1 + z_d)}{c} \frac{D_d D_s}{D_{ds}} \left[ \frac{1}{2} (\vec{\theta} - \vec{\beta})^2 - \psi(\vec{\theta}) \right] = t_{\text{geom}} + t_{\text{grav}}$$


**Fermat Principle for gravitational lensing**

**Images are located at points where the total time delay function is stationary**

$$\vec{\nabla}_{\theta} t(\vec{\theta}) = 0$$

# Gravitational lensing in a pill

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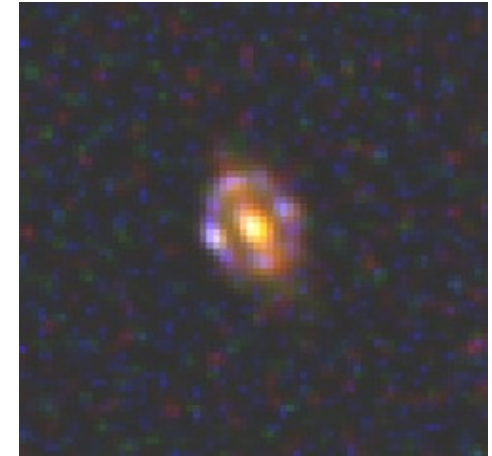
- Source magnification and distortion: main features of gravitational lensing



Credit: HST, with red ovals added by Sarah Bridle



NASA/CXC/Univ of Michigan/R.C.Reis et al;  
Optical: NASA/STScI



© MPIA / Arjen van der Wel

Lensing conserves surface brightness

Flux  $F =$  surface brightness  $\times$  solid angle

$$\text{Magnification} = F_{\text{observed}} / F_{\text{intrinsic}} = d\Omega_{\text{observed}} / d\Omega_{\text{intrinsic}}$$

Liouville's Theorem

# Gravitational lensing in a pill

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- Jacobian matrix for gravitational lensing:

$$\mathcal{A}(\boldsymbol{\theta}) = \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{\theta}} = \left( \delta_{ij} - \frac{\partial^2 \psi(\boldsymbol{\theta})}{\partial \theta_i \partial \theta_j} \right) = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}$$

where  $\gamma_1$  and  $\gamma_2$  are the two components of **shear**

$$\gamma \equiv \gamma_1 + i\gamma_2 = |\gamma| e^{2i\varphi}$$

$$\gamma_1 = \frac{1}{2}(\psi_{,11} - \psi_{,22}) \quad \gamma_2 = \psi_{,12}$$

Magnification in terms of  $\kappa$  and  $\gamma$  is:

$$\mu = \frac{1}{\det \mathcal{A}} = \frac{1}{(1 - \kappa)^2 - |\gamma|^2}$$

where  $\kappa$  is the **dimensionless surface mass density** (a.k.a. **convergence**)

$$\kappa(\boldsymbol{\theta}) = \frac{\Sigma(D_d \boldsymbol{\theta})}{\Sigma_{\text{cr}}}$$

and  $\Sigma_{\text{cr}}$  is the **critical surface mass density**

$$\Sigma_{\text{cr}} = \frac{c^2}{4\pi G} \frac{D_s}{D_d D_{\text{ds}}}$$

# Gravitational lensing in a pill

- Source magnification:

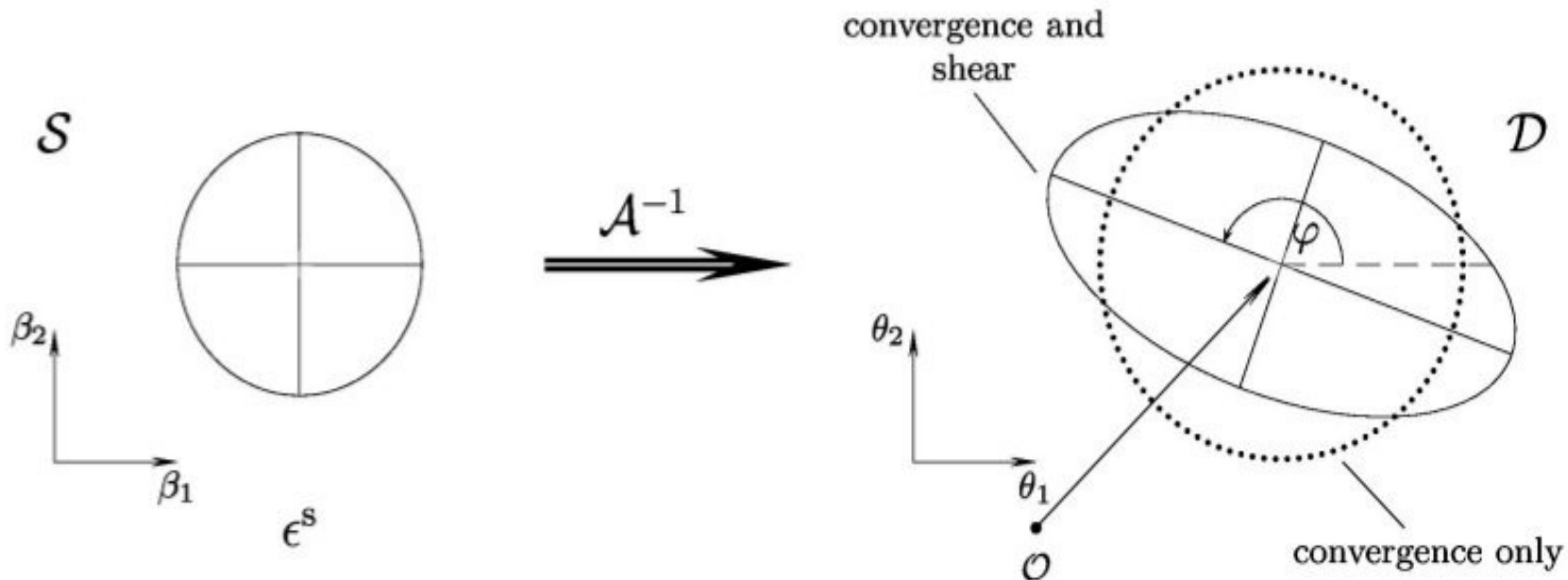
Magnification factor is

$$\mu(\boldsymbol{\theta}) = \frac{1}{\det \mathcal{A}(\boldsymbol{\theta})}$$

- $\mu > 0$  : positive parity
- $\mu < 0$  : negative parity (mirror image of source)
- $\det A = 0$  : critical points/curves

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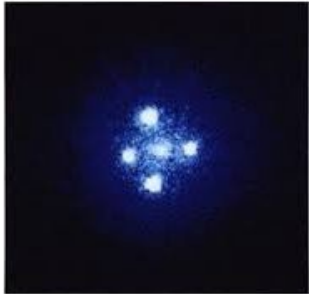
- Source distortion:



Credit: M. Bradac

# Gravitational lensing in a pill

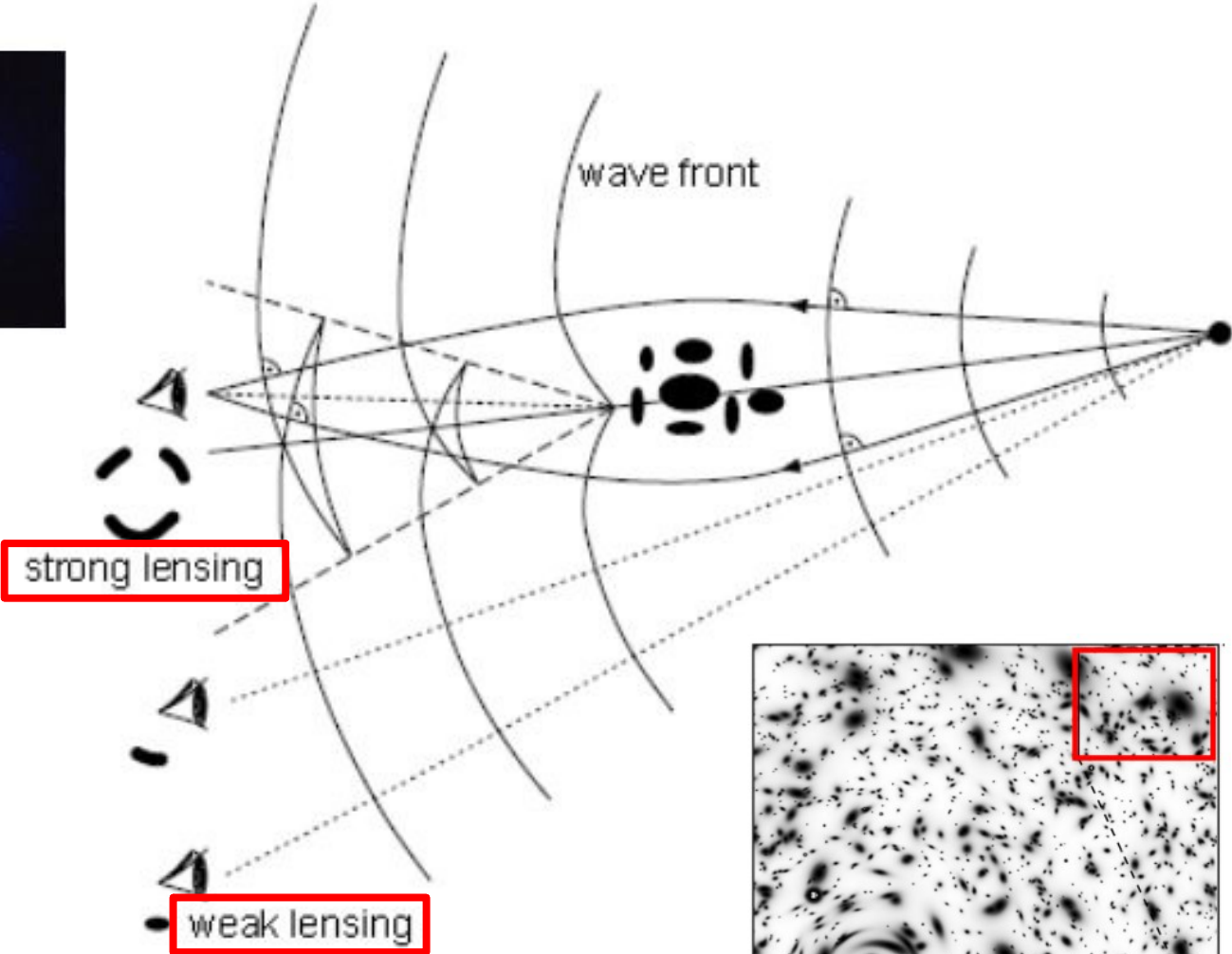
- Different regimes of gravitational lensing:



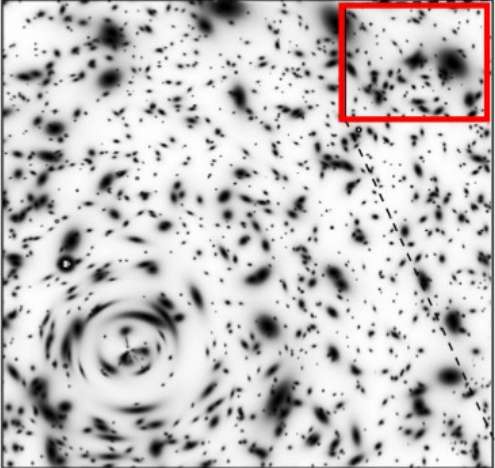
multiple images

time delays between images

images distorted into rings/arcs



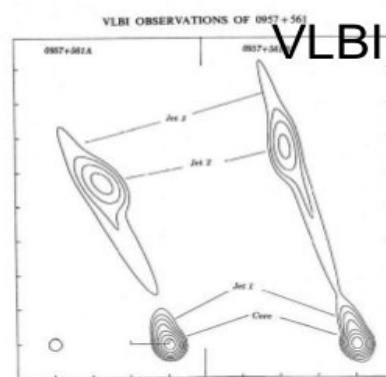
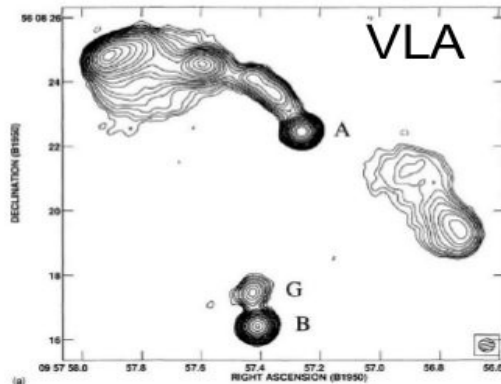
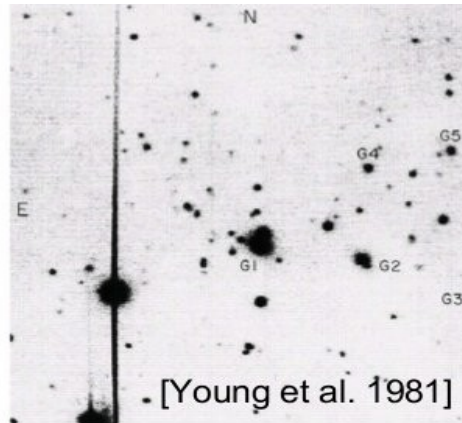
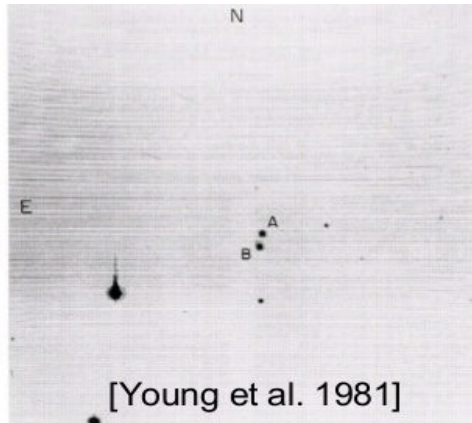
weak distortions of singly imaged background sources



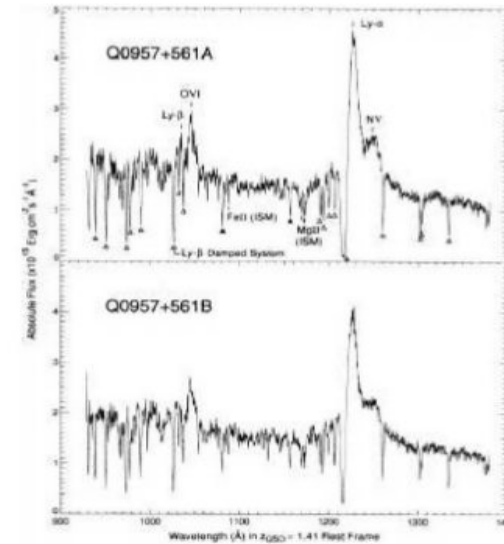
[Mellier 1999]

# Gravitational lensing in a pill

- F.Zwicky (1937): multiple images can be detected if one consider deflector as more massive than stars, e.g. galaxies
- Walsh, Carswell and Weymann (1979) – first detection of strongly lensed system: double quasar QSO 0957+561A,B



## The gravitational lens 0957+561



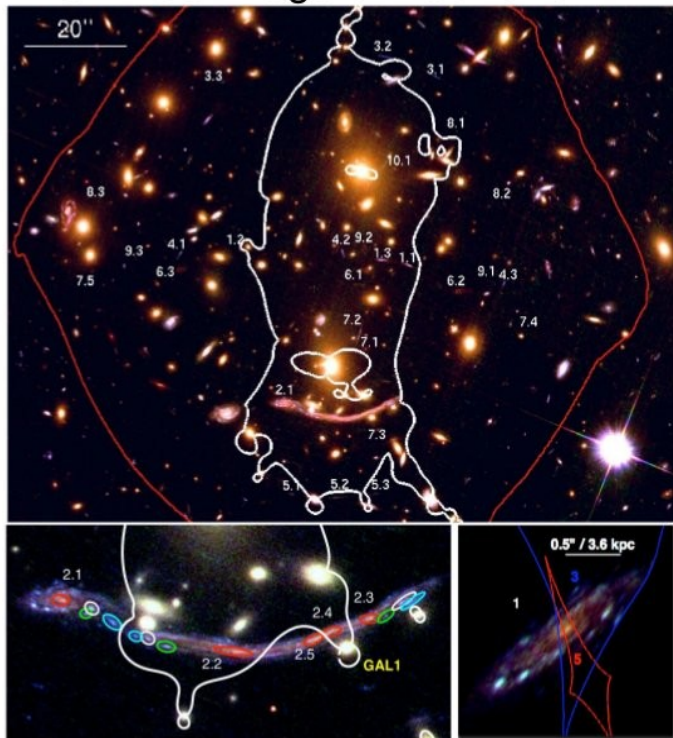
Michalitsianos et al. 1997

**identical spectra!**

# Gravitational lensing in a pill

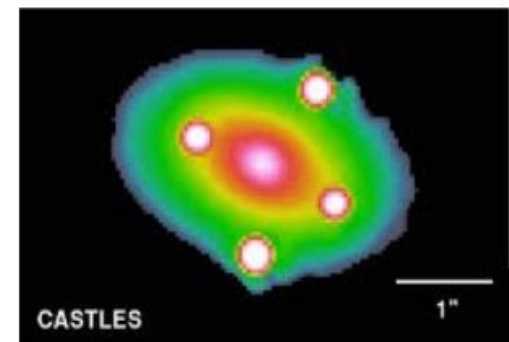
- Lynds and Petrosian (1986), Soucail (1987) – first detection of a giant arcs around two galaxy clusters: Abell 307

HST ACS image of Abell 370:



[Richard et al. 2010] <sup>16</sup>

- Irwin et al. (1989) – first detection of microlensing effect: QSO2237+0305

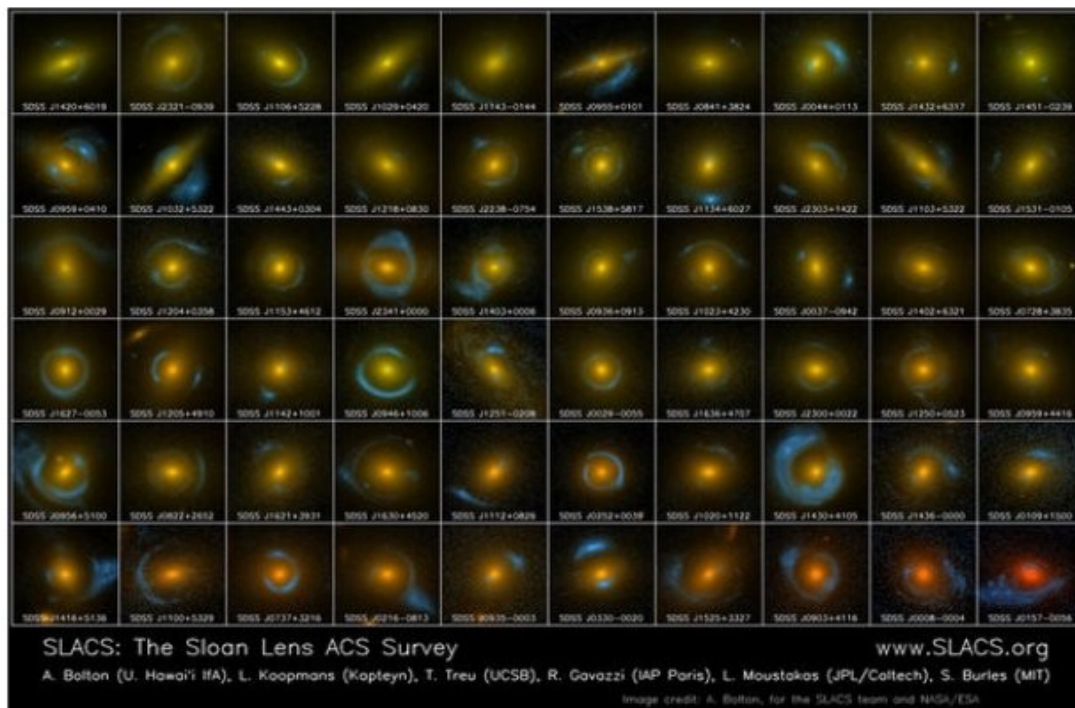


- Tyson, Valdes and Wenk (1990) – first detection of a weak lensing effect

# Gravitational lensing in a pill

- 1978-1992 – only 11 strong lensing systems was discovered
- Now – era of massive galactic surveys

searches concentrated on sources !



- Radio, optical imaging and spectroscopic surveys in recent years have led to an explosion in the discovery of lens systems

- Hundreds of strong lens systems now known

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“there is no great chance of observing this phenomenon” (Einstein 1936)



# Gravitational lensing in a pill

Sloan Lens ASC Survey **SLACS**

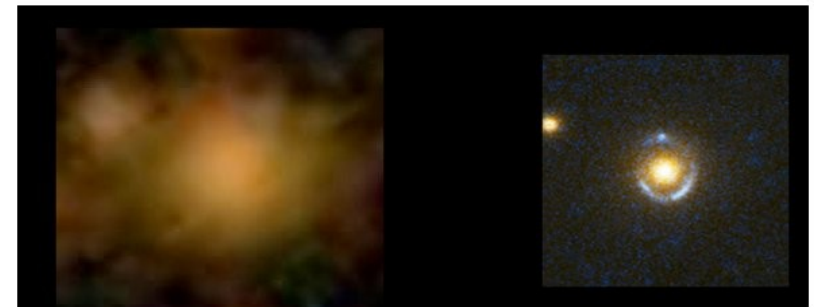
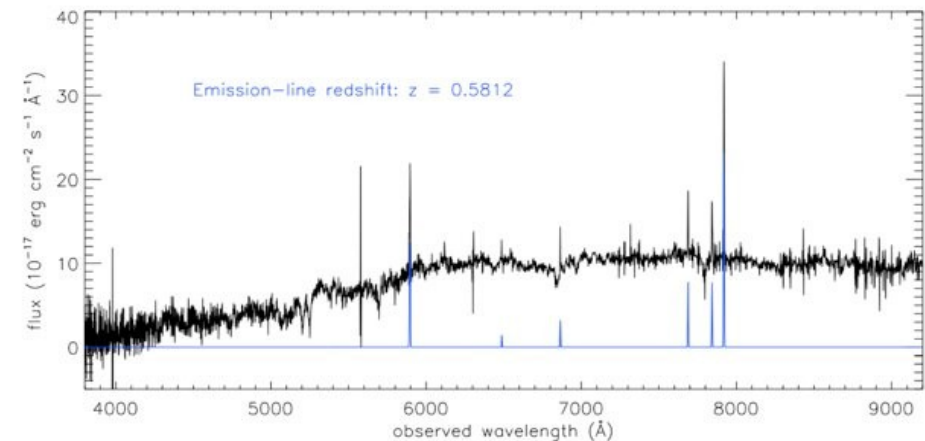
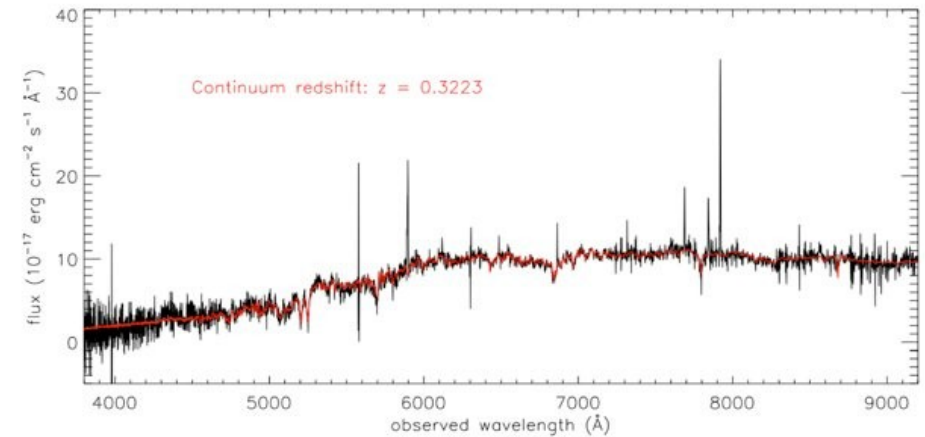
BOSS emission-line lens survey **BELLS**

spectroscopic surveys

lens candidates selected  
from SDSS/BOSS data

## Idea:

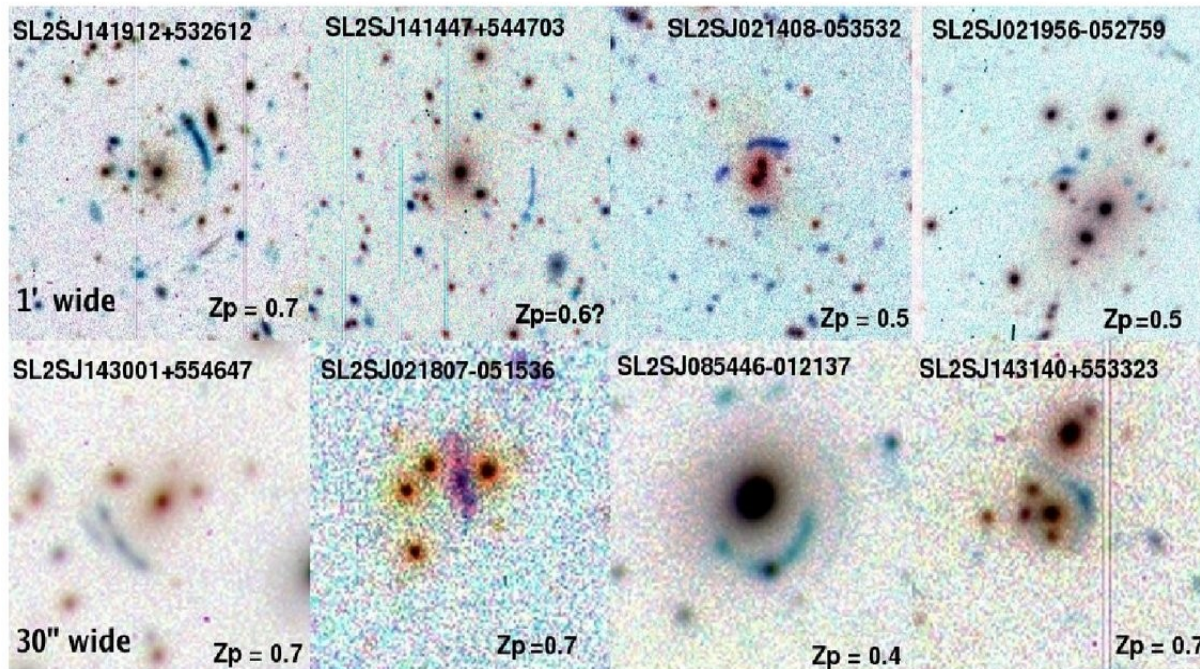
- looking for the presence of emission lines at redshifts higher than that of the target galaxy
- + HST ACS follow-up imaging



# Gravitational lensing in a pill

## Strong Lensing Legacy Survey **SL2S** from Canada France Hawaii Telescope Legacy Survey (CFHTLS)

- targets: massive red galaxies – **homogeneous sample**
  - fully automated software (*RingFinder*) looking for tangentially elongated blue features around
- [Gavazzi et al. 2014]



Early-type galaxies (ETGs)  
are more likely to serve  
as intervening galaxies:

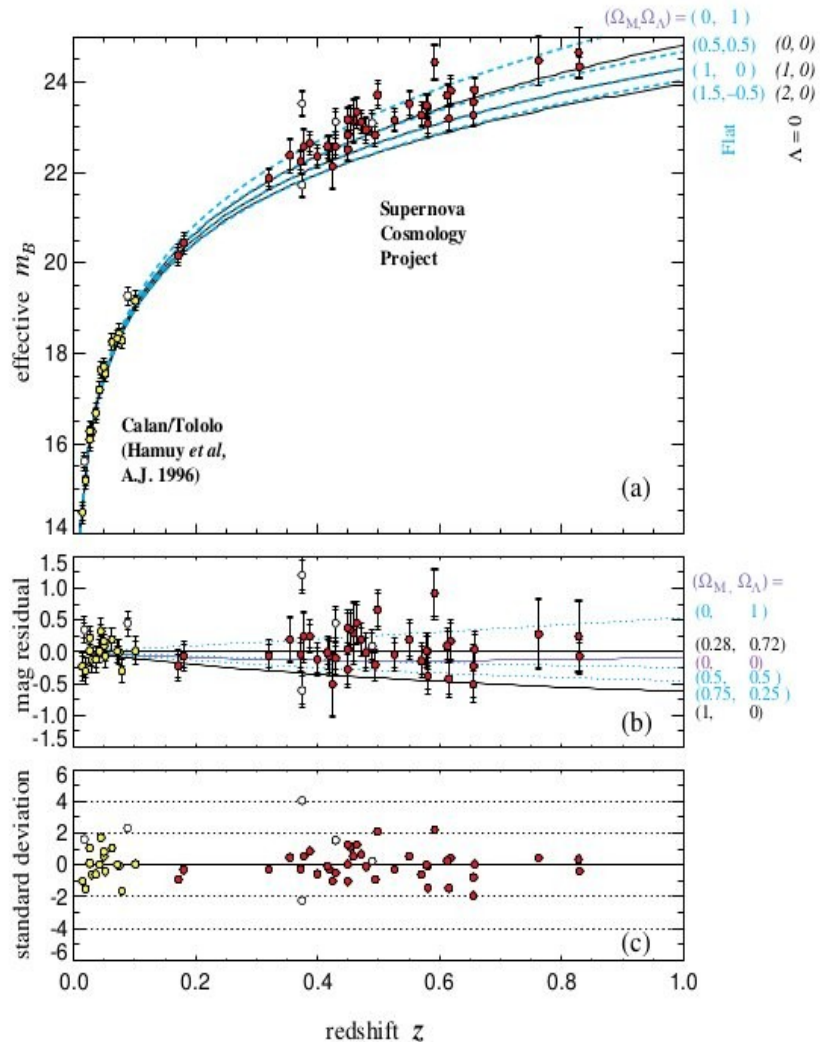
they contain most of  
the stellar mass of  
the Universe which affects  
statistics of gravitational  
lensing phenomenon

# Gravitational lensing as a tool in cosmology

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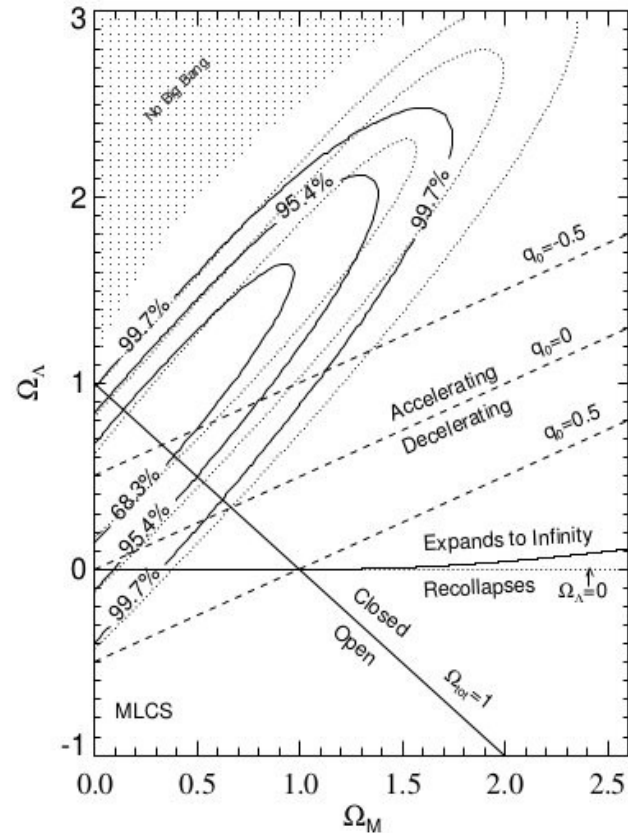
# The nature of dark energy

- Observational fact: present accelerating expansion of the Universe observed in Hubble diagrams from SNIa surveys



$$\Omega_\Lambda = 0.71 \pm 0.05 \quad (1\sigma)$$

$$\Omega_M = 0.29 \pm 0.05$$



Supernova Cosmology Project (Perlmutter et al. 1999)

High-z Supernova Search Team (Riess et al. 1998)

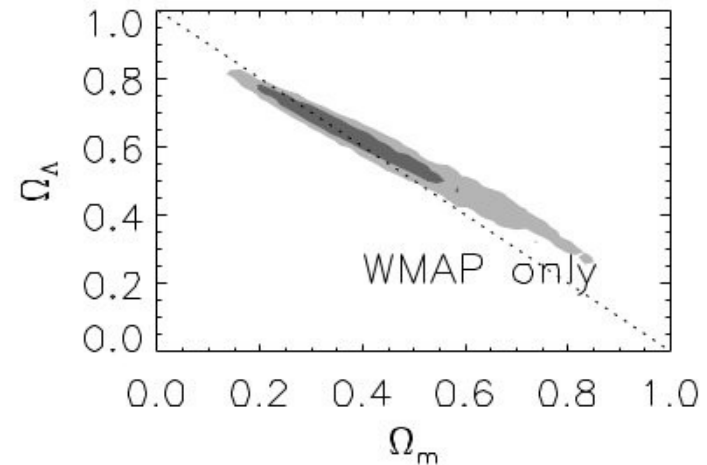
# The nature of dark energy

- SNIa results confirmed by independent estimates of the amount of baryons and cold dark matter:

$$\Omega_k = 1 - \Omega_M - \Omega_\Lambda$$

**spatially flat Universe**

First-Year WMAP data (Eisenstein et al. 2005)



First BAO measurements (Spergel et al. 2003)

TABLE 1

SUMMARY OF PARAMETER CONSTRAINTS FROM LRGs

Parameter	Constraint
$\Omega_m h^2$	$0.130(n/0.98)^{1.2} \pm 0.011$
$D_V(0.35)$	$1370 \pm 64$ Mpc (4.7%)
$R_{0.35} \equiv D_V(0.35)/D_M(1089)$	$0.0979 \pm 0.0036$ (3.7%)
$A \equiv D_V(0.35)(\Omega_m H_0^2)^{1/2}/0.35c$	$0.469(n/0.98)^{-0.35} \pm 0.017$ (3.6%)

NOTES.—We assume  $\Omega_b h^2 = 0.024$  throughout, but variations permitted by *WMAP* create negligible changes here. We use  $n = 0.98$ , but where variations by 0.1 would create  $1 \sigma$  changes, we include an approximate dependence. The quantity  $A$  is discussed in § 4.5. All constraints are  $1 \sigma$ .

TABLE 1

POWER-LAW  $\Lambda$ CDM MODEL PARAMETERS: *WMAP* DATA ONLY

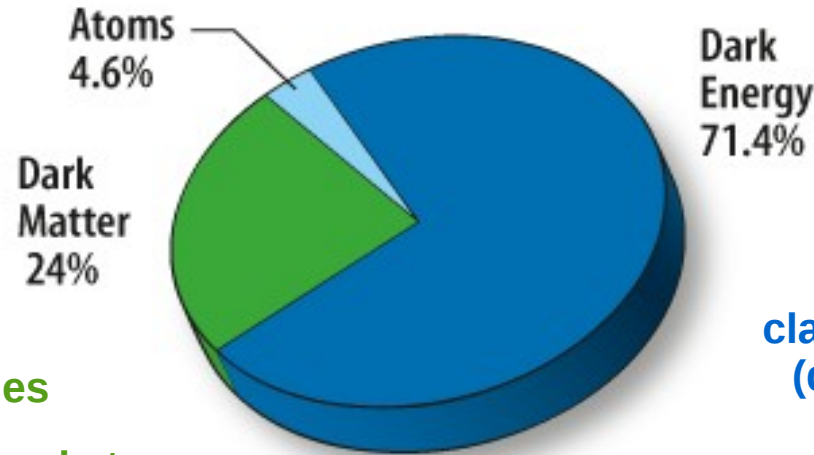
Parameter	Mean (68% Confidence Range)	Maximum Likelihood
Baryon density, $\Omega_b h^2$	$0.024 \pm 0.001$	0.023
Matter density, $\Omega_m h^2$	$0.14 \pm 0.02$	0.13
Hubble constant, $h$	$0.72 \pm 0.05$	0.68
Amplitude, $A$	$0.9 \pm 0.1$	0.78
Optical depth, $\tau$	$0.166^{+0.076}_{-0.071}$	0.10
Spectral index, $n_s$	$0.99 \pm 0.04$	0.97
$\chi_{\text{eff}}^2/\nu$		1431/1342

NOTE.—Fit to *WMAP* data only.

# The nature of dark energy

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- Cosmological consensus: most of the energy in the Universe exists in the form of the mysterious dark energy



flat rotation curves of galaxies  
gravitational lensing: weak and strong

classical cosmological tests  
(distance measurements)

TODAY

NASA/WMAP Science Team

modification of gravity at cosmological scales

exotic material component

new physics is needed

# The nature of dark energy

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- $\Lambda$ CDM model became a standard reference point in cosmology:

- FRW metric (homogeneous and isotropic spacetime)

$$ds^2 = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right]$$

- non-vanishing cosmological constant
- pressure-less matter including dark part of it

- The expansion rate in  $\Lambda$ CDM model can be parametrized in a very convenient way:

$$H^2(z) = H_0^2 [\Omega_m (1+z)^3 + \Omega_\Lambda]$$

$$\Omega_i = \frac{\rho_i}{\rho_c}$$

$$\Omega_{tot} = \sum_i \left( \frac{\rho_i}{\rho_c} \right) = 1$$

strong evidence for the spatially flatness of the Universe from observations

# The nature of dark energy

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- $\Lambda$ CDM model even best fitted to observations suffers however from several problems of fundamental nature:

fine tuning problem

discrepancy between facts and expectations

- One can heuristically assume that dark energy is described by hydrodynamical energy-momentum tensor with (effective) cosmic EoS:

$$w = 0 \quad \text{dust}$$

$$w = 1/3 \quad \text{radiation}$$

$$p = w\rho$$

$$w = -1$$

cosmological constant

- Time-varying EoS as a Taylor expansion over  $a(t)$  ( linear order ):

$$w(z) = w_0 + w_a \frac{z}{1+z}$$

If we think that dark matter has its origins in the evolving scalar field (quintessence), it would be natural to expect that the  $w$  coefficient should vary in time

**CPL parametrization**  
Chevalier&Polarski 2001, Linder 2003



# The nature of dark energy

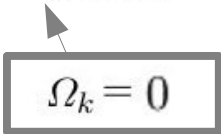
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- The nature of dark energy is still an open question
- We are left with the phenomenological approach based on upgrading observational fits of quantities parametrizing dark energy

**density parameters or coefficients in the cosmic EoS**

- The most general phenomenological form of the expansion rate is determined by a set of parameters:

$$H(t)^2 = H_0^2 \left[ \Omega_m a(t)^{-3} + \Omega_r a(t)^{-4} + \Omega_X a(t)^{-3(1+w_X)} + \Omega_k a(t)^{-2} \right]$$


$$\Omega_k = 0$$

**Technically speaking: testing cosmological models means to determine parameters from observables measured on extragalactic objects laying on cosmological distances**

# Distance measures

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- One of the very direct cosmological probes could be to test the distance-redshift relation  $D(z)$  (Hubble diagram)
- In non-Euclidean geometry one distinguishes three types of distances:

- **comoving distance**

$$r(z; \mathbf{p}) = c \int_0^z \frac{dz'}{H(z'; \mathbf{p})} = \frac{c}{H_0} \tilde{r}(z; \mathbf{p}) \quad \text{not measured directly}$$

- **luminosity distance**

$$D_L(z; \mathbf{p}) = (1 + z)r(z; \mathbf{p})$$

measured on objects with known luminosity  
– **standard candles**

- **angular diameter distance**

$$D_A(z; \mathbf{p}) = \frac{1}{1 + z} r(z; \mathbf{p})$$

measured on objects with known angular size  
– **standard rulers** (statistical and individual)

# Cosmological probes

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- Standard candles: **SN Ia**

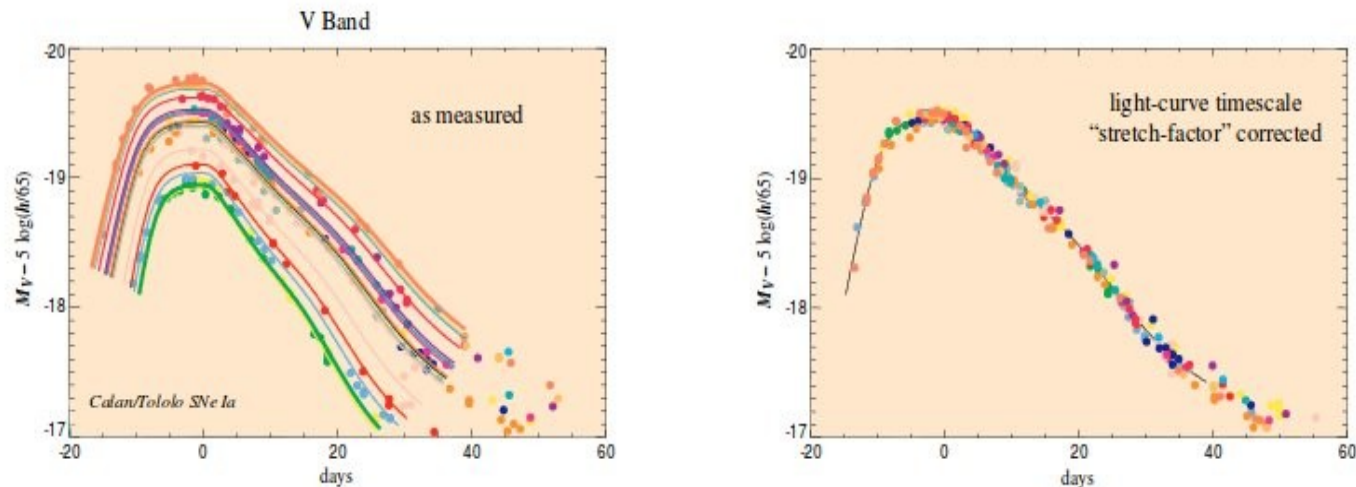
- bright enough to be detected in distant galaxies (up to  $z \sim 1.7$ )
- the most recent compilation of 557 SN Ia data known as Union2

Amanullah et al. 2010, Suzuki et al. 2011

- luminosity distance vs. redshift relation via distance modulus:

$$\mu := m - M = 5 \log_{10}(D_L(z; \mathbf{p})) + 25$$

**Standardizable** – luminosity correlated with duration and spectral features of the event



Perlmutter et al. 1998

# Cosmological probes

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- Standard(izable) candles available in the future:

## Other type of SNe (SNI-P)

- type II SNe are not as bright as the SN Ia but they are the most common type of SNe
- correlation between expansion velocities of the ejecta and bolometric luminosities in the plateau phase

## GRBs

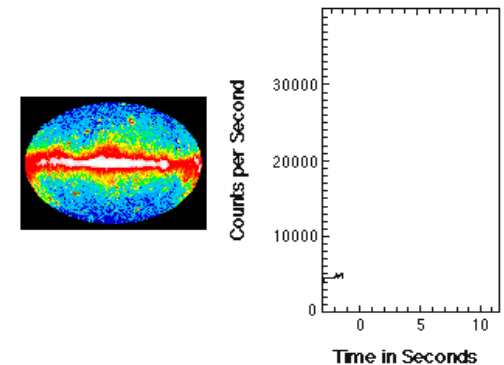
- detectable up to the redshift of  $z \sim 8$
- several suggestions to calibrate them by using correlations between various properties of the prompt emission and in some cases also the afterglow emission

## Gravitational-wave sources (standard sirens)

- the most promising source: inspiral and merger of a compact-object binaries consisting of neutron stars and/or black holes
- redshift and luminosity distance of the system is directly encoded in the waveform

Poznanski, Nugent  
& Filippenko 2010

Hamuy & Pinto 2002



Capozziello et al. 2012

Arabsalmani, Sahni  
& Saini 2013

Camera & Nishizawa 2013

Taylor & Gair 2012

# Cosmological probes

- Statistical standard rulers: **CMBR and BAO**

angular size of the radius of the sound horizon size at the decoupling epoch

## the sound horizon

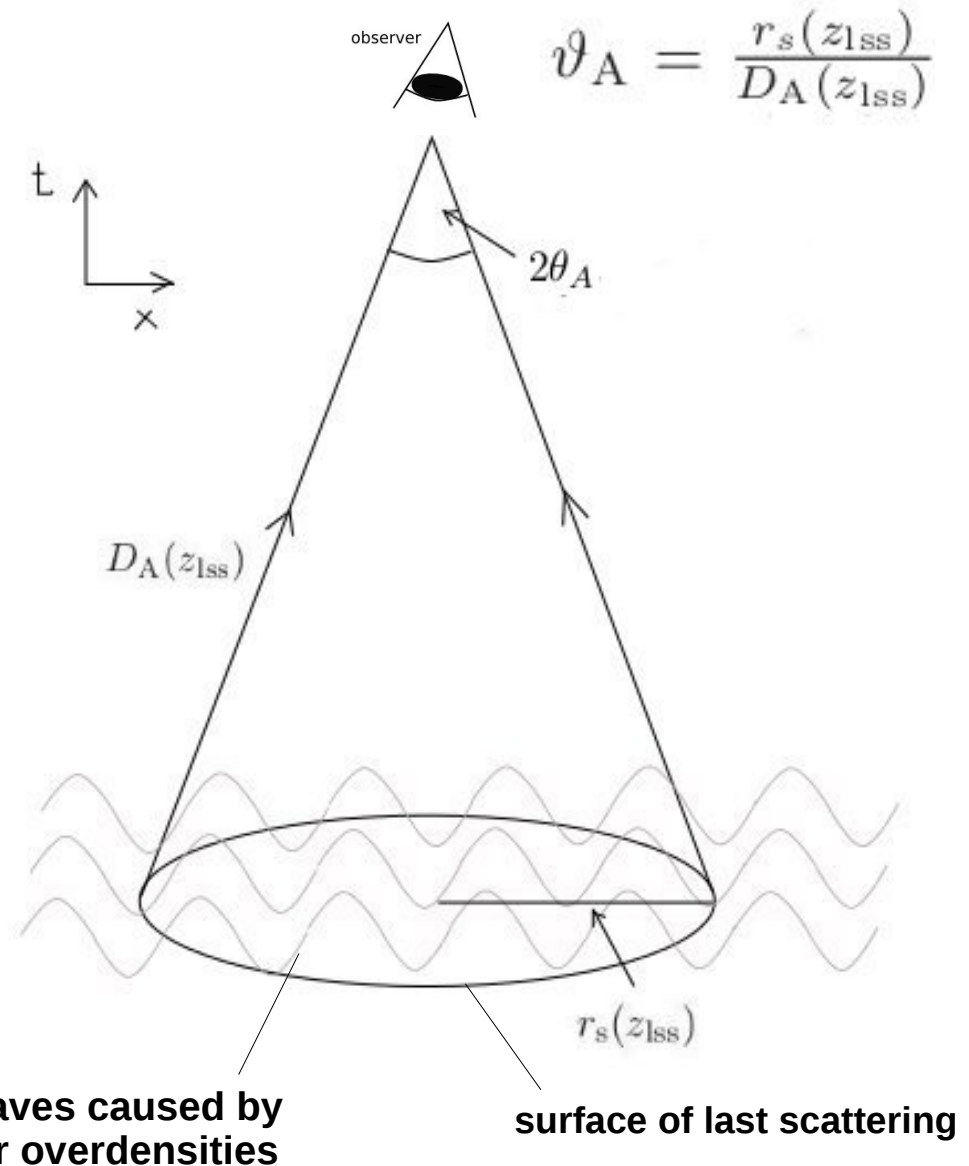
comoving distance travelled by a sound wave in the photon-baryon fluid by the time of decoupling ( $z \sim 1100$ )

$$r_s(z_{\text{lss}}) = \int_{z_{\text{lss}}}^{\infty} \frac{c_s dz}{H(z)}$$

depends on baryon and matter densities  
(known from CMBR measurements)

$$r_s(z_{\text{lss}}) = 146.8 \pm 1.8 \text{Mpc}$$

Komatsu et al. 2008



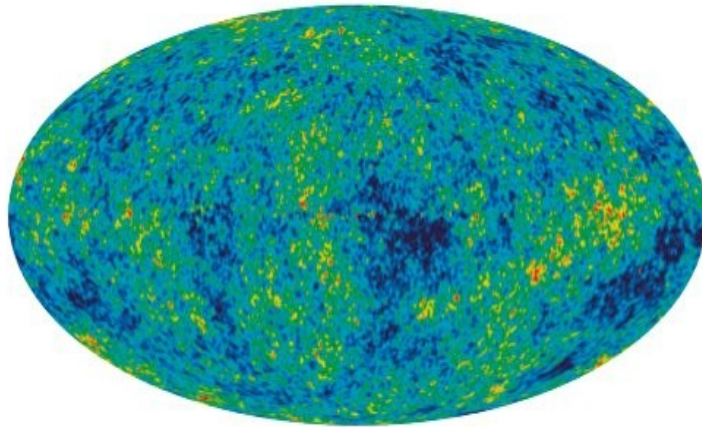
$$\vartheta_A = \frac{r_s(z_{\text{lss}})}{D_A(z_{\text{lss}})}$$

pressure waves caused by dark matter overdensities

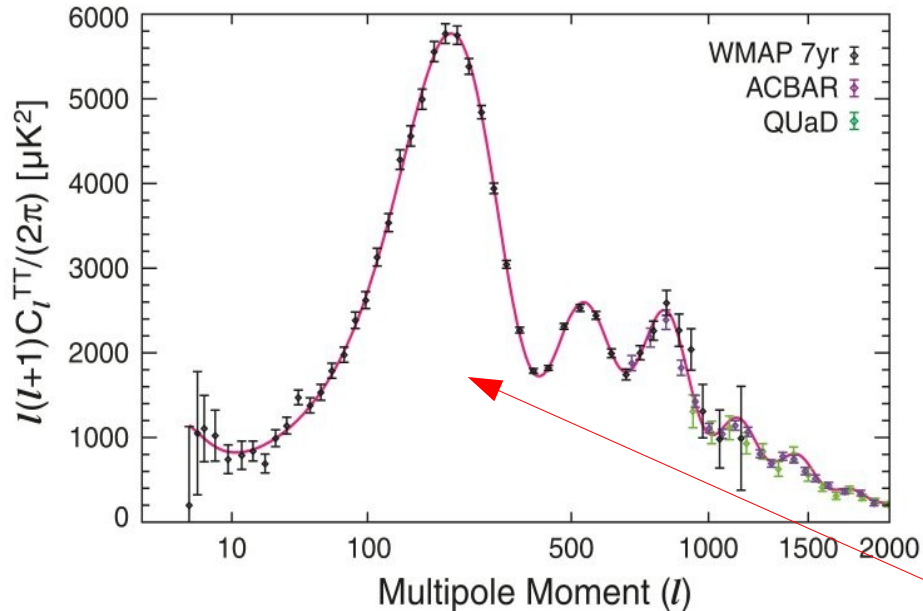
surface of last scattering

# Cosmological probes

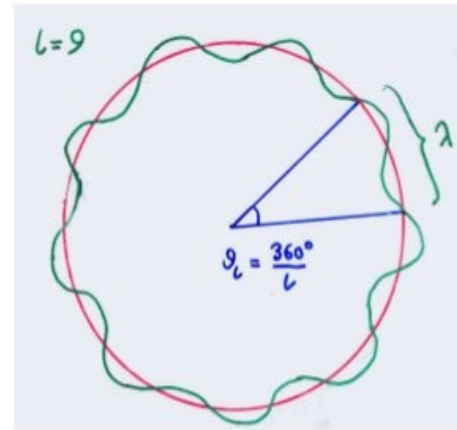
- **CMBR anisotropies** - the pattern of acoustic oscillations frozen in the CMB



WMAP 5-year  
-200 T(μK) +200



the angles on the sky are related to actual physical or comoving distances via the angular diameter distance



$$l \propto 1/\theta$$

different multipole numbers  $l$  correspond to different angular scales:

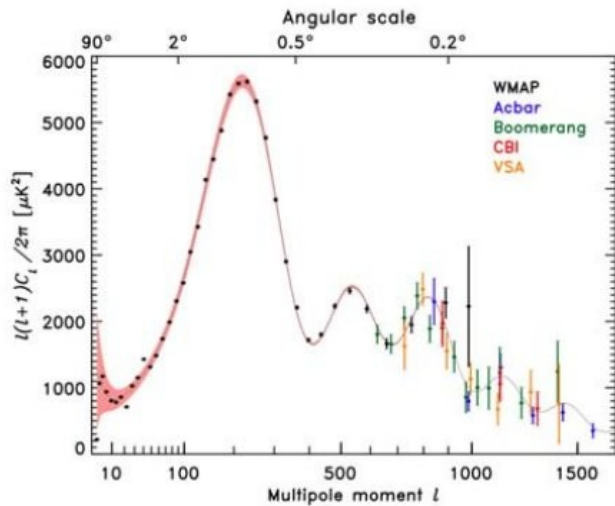
$$l \approx \frac{D_A(z_{lss})}{r_s}$$

**location of the first acoustic peak depends strongly on geometry and cosmology**

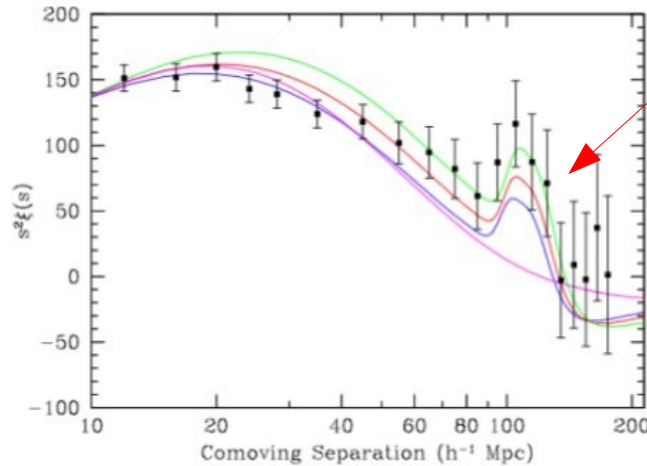
# Cosmological probes

- **BAO**

Besides producing the acoustic peaks of the CMBR, pressure waves reveal themselves in clustering properties of galaxies:

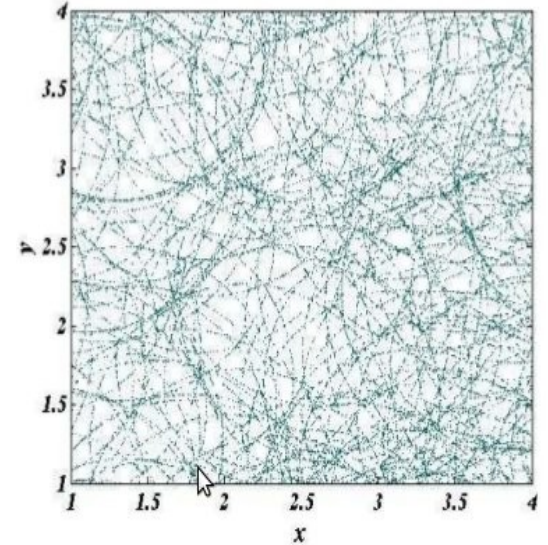


Hinshaw et al. 2003



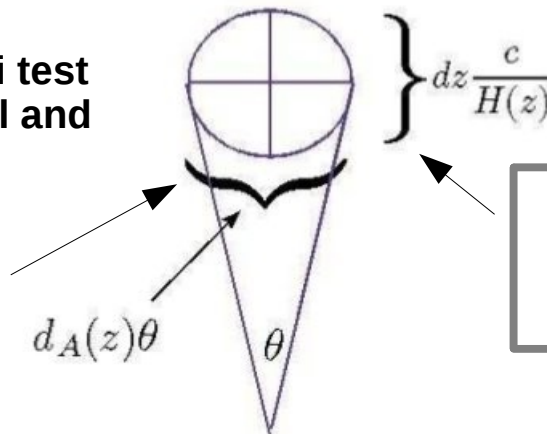
Eisenstein et al. 2005

a bump in the two-point correlation function in distribution of LRG (SDSS)



simple idea of the Alcock-Paczynski test - correlations measured in the radial and transverse direction:

$$d_A(z) = \frac{s_{\perp}}{\Delta\theta(1+z)}$$



$$H(z) = \frac{c\Delta z}{s_{\parallel}(z)}$$

Bassett, Hlozek et al. 2009

# Cosmological probes

- Individual standard rulers:

## Ultra compact radio sources

- standard ruler – size of the central region of AGNs
- evolution free sample – morphology depends only on the nature of the central engine controlled by a limited number of physical parameters: mass of the central black hole, magnetic field,
- accretion rate, angular momentum (possibly)

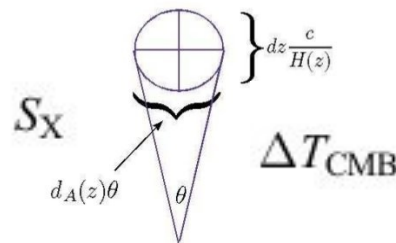
## Double-sided radio sources (FR IIb radio galaxies)

- standard ruler – physical size of the radio bridge structure
- evolution of structure is linear with time (older are bigger)

## Galaxy clusters

- combined X-ray+SZ data
- distance inferred from AP test (assumption: symmetrical spherical shape of the cluster)

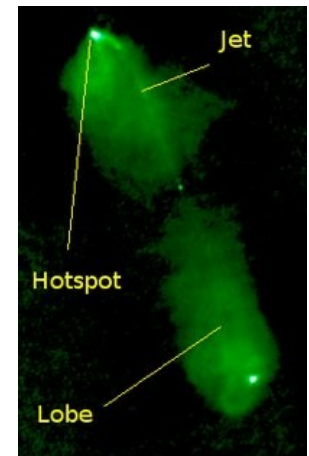
$$D_A \equiv dl/d\theta$$



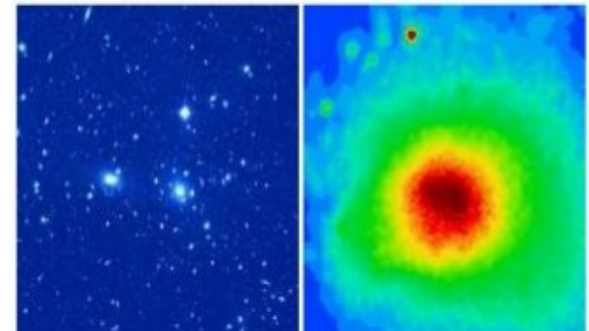
Bonamente et al. 2006

Gurvits, Kellermann,  
Frey 1998

Gurvits 1994



Daly 1994, 2009

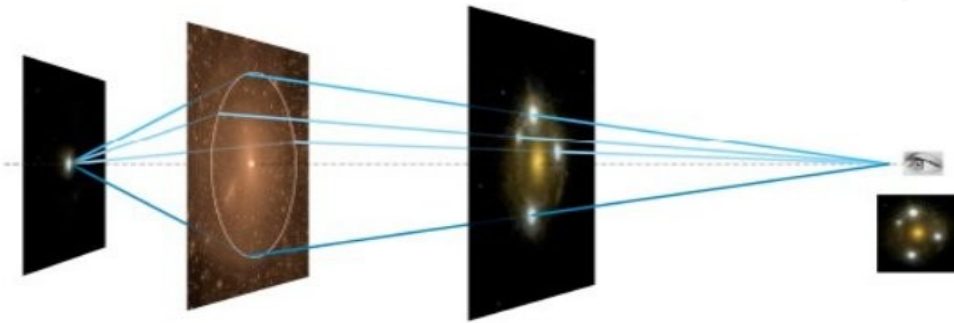


wide range of redshifts



# SL as standard(izable) rulers – our method

- **The idea:** image separations in the system depend on angular diameter distances to the lens and to the source, which in turn are determined by background cosmology



majority of cases the lens is a late-type E/SO galaxy

$$\rho(r) = \frac{\sigma_{SIS}^2}{2\pi G} \frac{1}{r^2}$$

**SIS model – the simplest realistic case**

**Einstein radius defines characteristic angular scale for the lens:**

$$\theta_E = 4\pi \frac{D_{ls}}{D_s} \frac{\sigma_{SIS}^2}{c^2}$$

distance from the lens to the source →  $D_{ls}$

from angular image separations (astrometry) ←  $\theta_E$

←  $D_s$  distance from observer to the source

←  $\sigma_{SIS}^2$  one-dimensional velocity dispersion in lensing galaxy (spectroscopy)

$\sigma_{SIS}$  lens velocity dispersion is well approximated by  $\sigma_0$  - central stellar velocity dispersion (see eg. Grillo et al. 2008)

# Strong lenses as standard(izable) rulers

observable: distance ratio

$$D^{th}(z_l, z_s, p) \rightarrow \frac{D_s}{D_{ls}} = \frac{4\pi\sigma_0^2}{c^2\theta_E} \leftarrow D^{obs}$$

$$D_A(z; p) = \frac{c}{H_0} \frac{1}{1+z} \int_0^z \frac{dz'}{h(z'; p)}$$

this opens a possibility to constraining the cosmological model provided that we have good knowledge of the lens model (i.e. SIS model for elliptical galaxies)

growing evidence for homologous structure of early type galaxies supporting reliability of SIS assumption

gets canceled in the distance ratio

Koopmans et al. 2006, 2009

method is independent of the Hubble constant's value and is not affected by dust absorption or source evolutionary effects

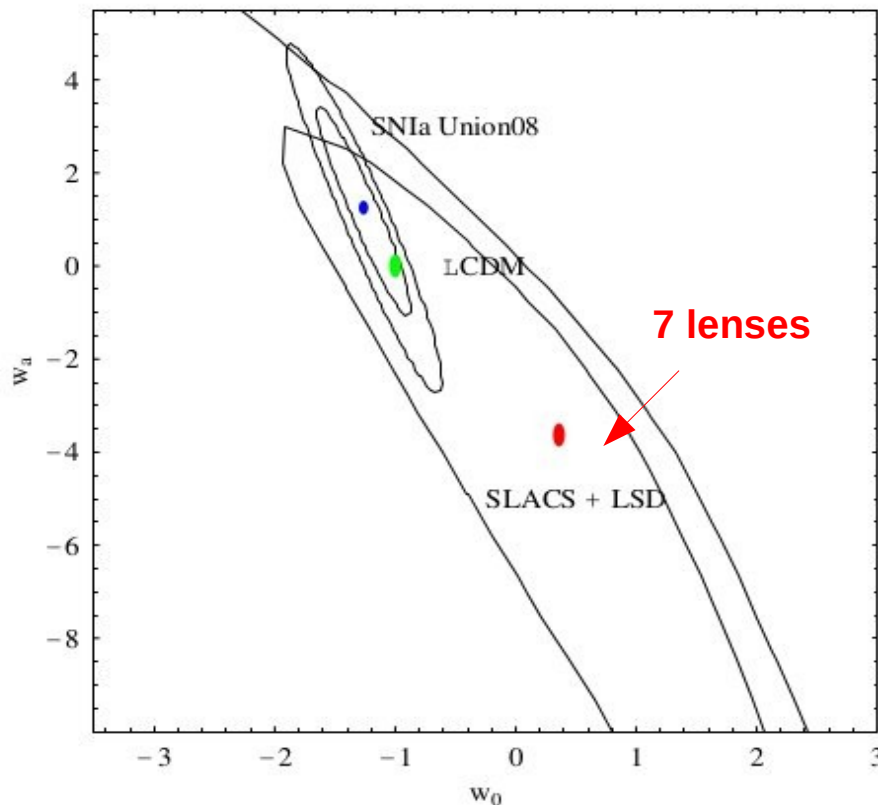
# Strong lenses as standard(izable) rulers

- Cosmological model parameters (coefficients in the equation of state) are estimated by minimizing following chi-square function:

$$\chi^2(\mathbf{p}) = \sum_i \frac{(\mathcal{D}_i^{\text{obs}} - \mathcal{D}_i^{\text{th}}(\mathbf{p}))^2}{\sigma_{\mathcal{D},i}^2}$$

$$\mathcal{D}^{\text{th}}(z_l, z_s; \mathbf{p}) = \frac{D_s(\mathbf{p})}{D_{\text{ls}}(\mathbf{p})} = \frac{\int_0^{z_s} \frac{dz'}{h(z'; \mathbf{p})}}{\int_{z_l}^{z_s} \frac{dz'}{h(z'; \mathbf{p})}}$$

$$\mathcal{D}^{\text{obs}} = \frac{4\pi\sigma_0^2}{c^2\theta_E^2}$$



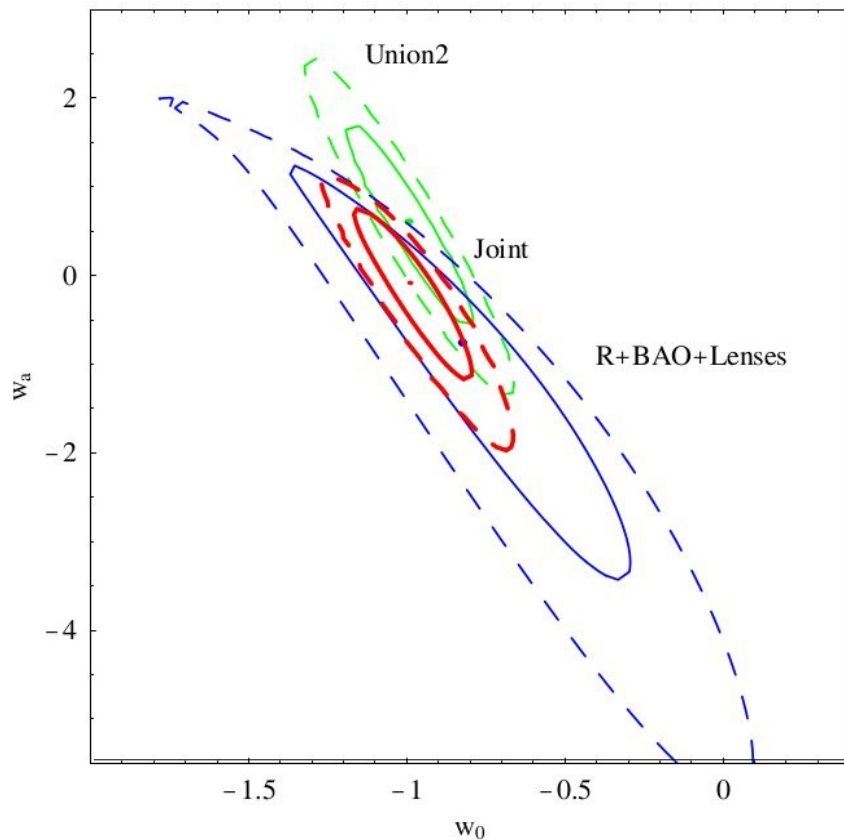
Best fits (dots) and (68%, 98%) confidence regions for CPL parameters in cosmic equation of state obtained from SLACS and SLD sample of lenses and Union08 SNIa data.

Biesiada, AP, Malec, 2010

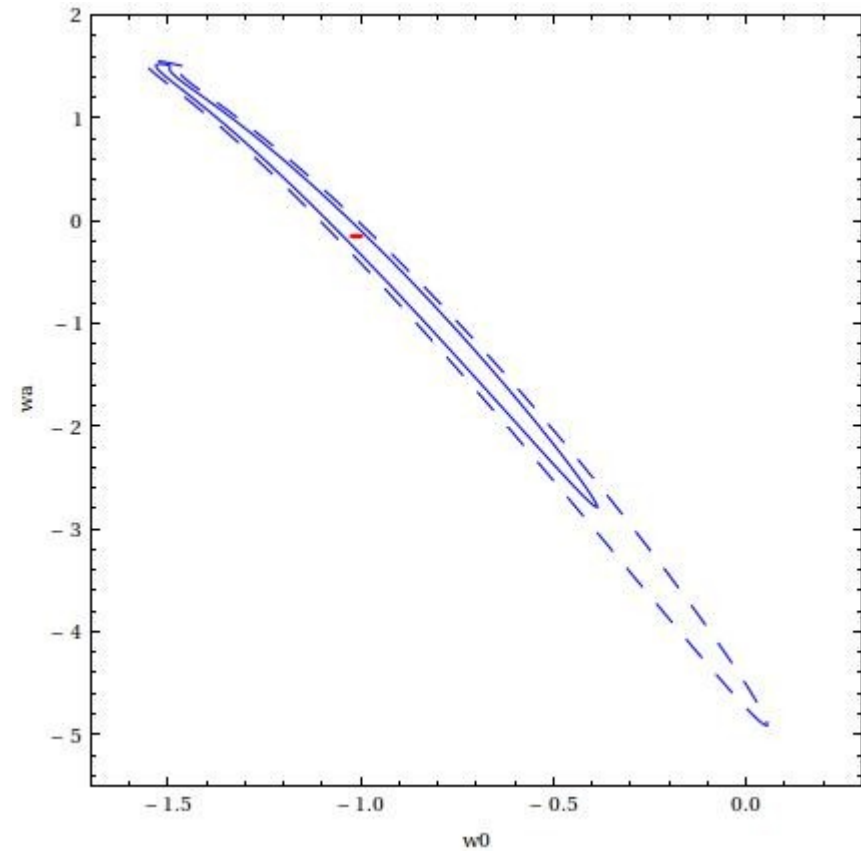
# Strong lenses as standard(izable) rulers

- A joint analysis of CPL model on rulers (R+BAO+Lenses):

20 strong lensing systems with good spectroscopic measurements of central dispersions from the SLACS and LSD surveys



Biesiada, Malec, AP, 2011



SLACS+LSD+SL2S surveys and taking into account the evolution of the total mass density slope inside the Einstein radius for each of the lens galaxy

Biesiada, Gavazzi & AP, COSMO Probes, Lausanne 2013

Ruff, Gavazzi et al. 2011

# Strong lenses as standard(izable) rulers

- **First step forward:** new updated compilation of 118 lenses from SLD, SLACS, BELLS and SL2S catalogues.

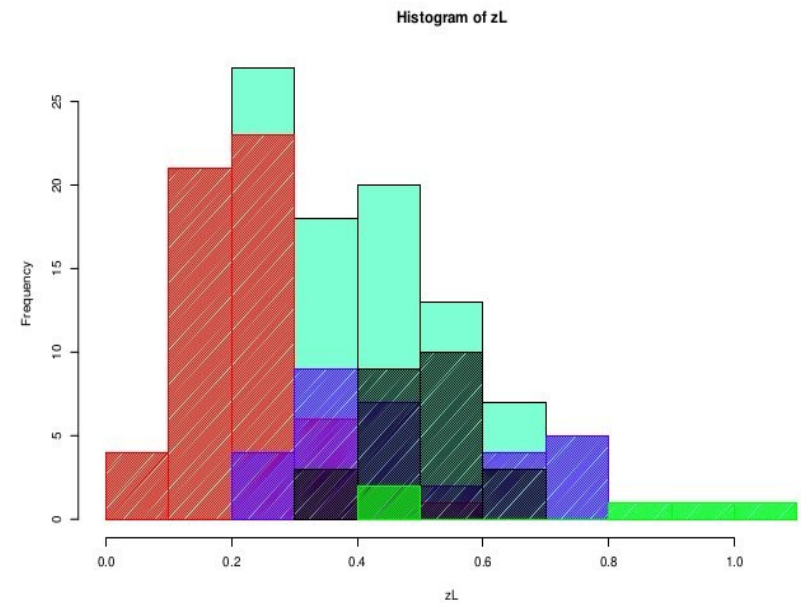
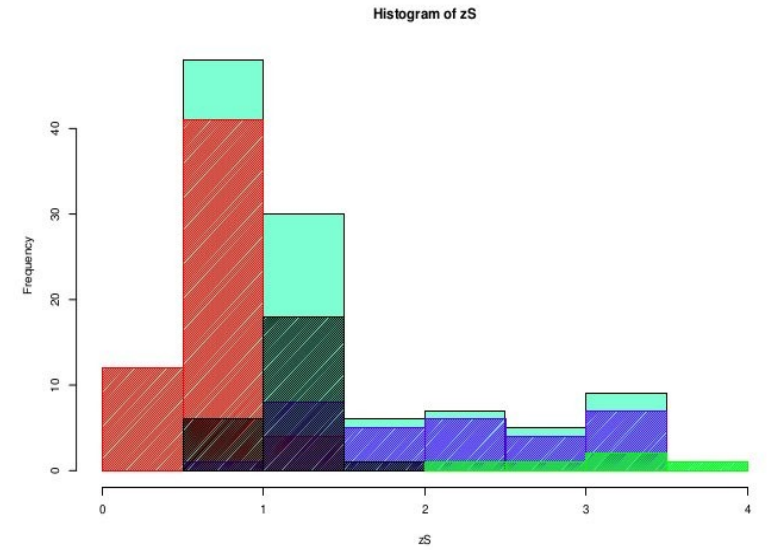
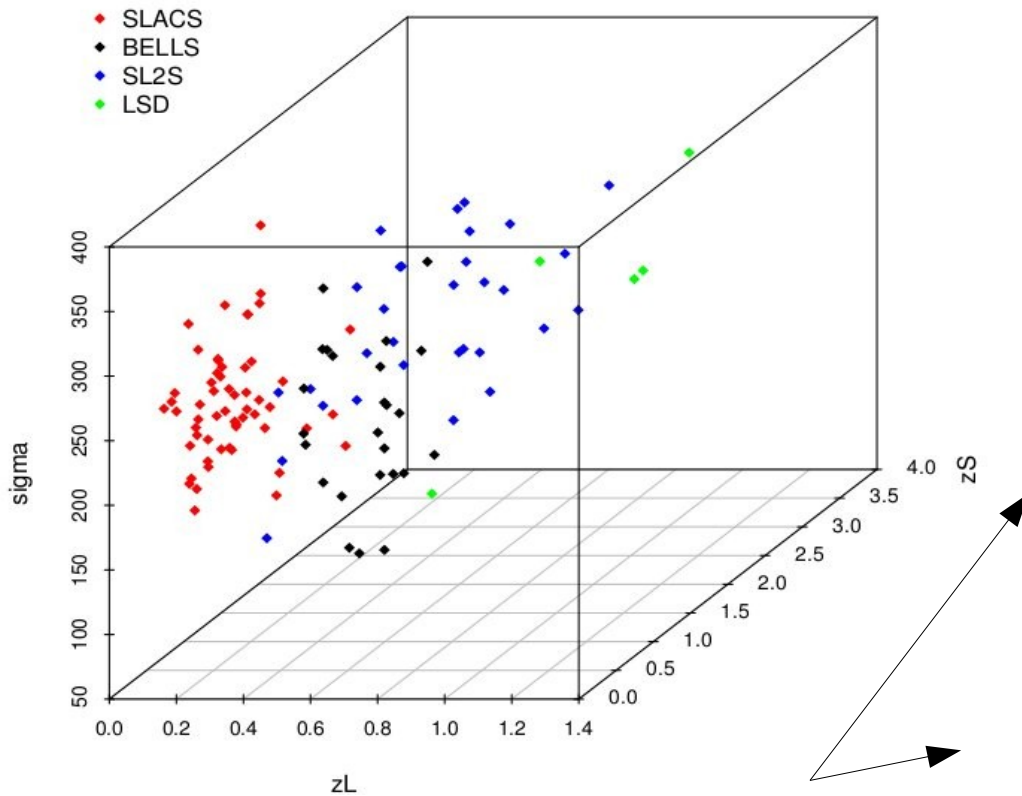


Fig. 1. Different surveys have the following median values of the lens redshifts: SLACS –  $z_l = 0.215$ , BELLS –  $z_l = 0.517$ , LSD –  $z_l = 0.81$  and SL2S –  $z_l = 0.456$ . The above values refer to lenses used by us. SL2S survey – an ongoing one – is particularly promising for the future since it already reached the maximum lens redshift of  $z_l = 0.8$ .

# Strong lenses as standard(izable) rulers

- **First step forward:** new updated compilation of 118 lenses from SLD, SLACS, BELLS and SL2S catalogues:

TABLE 1  
COMPILATION OF STRONG LENSING SYSTEMS

Name	$z_l$	$z_s$	$\sigma_{ap}$ [km/s]	$\theta_E$ ["]	survey	$\theta_{ap}$ ["]	$\theta_{eff}$ ["]	$\sigma_0$ [km/s]
J0151+0049	0.517	1.364	219±39	0.68	BELLS	1	0.89	226±40
J0747+5055	0.438	0.898	328±60	0.75	BELLS	1	1.24	334±61
J0747+4448	0.437	0.897	281±52	0.61	BELLS	1	2.87	277±51
J0801+4727	0.483	1.518	98±24	0.49	BELLS	1	0.57	103±25
J0830+5116	0.53	1.332	268±36	1.14	BELLS	1	1.1	274±37
J0944-0147	0.539	1.179	204±34	0.72	BELLS	1	1.35	207±35
J1159-0007	0.579	1.346	165±41	0.68	BELLS	1	0.99	170±42
J1215+0047	0.642	1.297	262±45	1.37	BELLS	1	1.42	266±46
J1221+3806	0.535	1.284	187±48	0.7	BELLS	1	0.93	193±49
J1234-0241	0.49	1.016	122±31	0.53	BELLS	1	1.61	123±31
J1318-0104	0.659	1.396	177±27	0.68	BELLS	1	1.06	182±28
J1337+3620	0.564	1.182	225±35	1.39	BELLS	1	1.6	227±35
J1349+3612	0.44	0.893	178±18	0.75	BELLS	1	2.03	178±18
J1352+3216	0.463	1.034	161±21	1.82	BELLS	1	1.35	164±21
J1522+2910	0.555	1.311	166±27	0.74	BELLS	1	1.08	170±28
J1541+1812	0.56	1.113	174±24	0.64	BELLS	1	0.59	183±25
J1542+1629	0.352	1.023	210±16	1.04	BELLS	1	1.45	213±16
J1545+2748	0.522	1.289	250±37	1.21	BELLS	1	2.65	247±37
J1601+2138	0.544	1.446	207±36	0.86	BELLS	1	0.63	217±38
J1611+1705	0.477	1.211	109±23	0.58	BELLS	1	1.33	111±23
J1631+1854	0.408	1.086	272±14	1.63	BELLS	1	2.07	272±14
J1637+1439	0.391	0.874	208±30	0.65	BELLS	1	0.89	215±31

# Strong lenses as standard(izable) rulers

- **Second step forward:** generalization of the SIS model to spherically symmetric power-law mass distribution.

$$\rho \sim r^{-\gamma}$$

mass inside the Einstein radius:

$$M_{lens} = \frac{c^2}{4G} \frac{D_s D_l}{D_{ls}} \theta_E^2$$

[Schneider et al.1992]

formula for the einstein radius  
in general spherically symmetric  
power-law lens mass distribution

$$\theta_E = 4\pi \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left( \frac{\theta_E}{\theta_{ap}} \right)^{2-\gamma} f(\gamma)$$

dynamical mass inside the aperture  
projected to lens plane:

$$\begin{aligned} M_{dyn} &= \frac{\pi}{G} \sigma_{ap}^2 R_E \left( \frac{R_E}{R_{ap}} \right)^{2-\gamma} f(\gamma) \\ &= \frac{\pi}{G} \sigma_{ap}^2 D_l \theta_E \left( \frac{\theta_E}{\theta_{ap}} \right)^{2-\gamma} f(\gamma) \end{aligned}$$

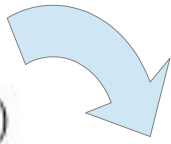
$$\begin{aligned} f(\gamma) &= -\frac{1}{\sqrt{\pi}} \frac{(5-2\gamma)(1-\gamma)}{3-\gamma} \frac{\Gamma(\gamma-1)}{\Gamma(\gamma-3/2)} \\ &\times \left[ \frac{\Gamma(\gamma/2-1/2)}{\Gamma(\gamma/2)} \right]^2 \end{aligned}$$

[Koopmans et al. 2005]


# Strong lenses as standard(izable) rulers

- Making our sample more uniform – correction of the velocity dispersion.

new observable

$$\theta_E = 4\pi \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left( \frac{\theta_E}{\theta_{ap}} \right)^{2-\gamma} f(\gamma)$$


$$\mathcal{D}^{obs} = \frac{c^2 \theta_E}{4\pi \sigma_{ap}^2} \left( \frac{\theta_{ap}}{\theta_E} \right)^{2-\gamma} f^{-1}(\gamma)$$



we need to transform all velocity dispersions measured within an aperture to those, measured within circular aperture of radius  $R_{eff} / 2$

$$\sigma_0 = \sigma_{ap} (\theta_{eff} / (2\theta_{ap}))^{-0.04}$$

uncertainties of the effective radius contribute less than 1% to the uncertainty of  $\sigma_0$

[Jorgensen et al.1995]

this operation makes our observable more homogeneous for the sample of lenses located at different redshifts



# Strong lenses as standard(izable) rulers

- Monte Carlo (CosmoMC package) simulations of the posterior likelihood

$$\mathcal{L} \sim \exp(-\chi^2/2)$$

$$\chi^2 = \sum_{i=1}^{118} \left( \frac{\mathcal{D}^{th}(z_{l,i}, z_{s,i}; \mathbf{P}, \gamma) - \mathcal{D}^{obs}(\sigma_{0,i}, \theta_{E,i})}{\Delta \mathcal{D}_i^{obs}} \right)^2$$

$\gamma$  taken as a free parameter !

5%  
SLACS Team

$$\delta \mathcal{D} = \frac{\Delta \mathcal{D}}{\mathcal{D}} = \sqrt{4(\delta \sigma_{ap})^2 + (1 - \gamma)^2 (\delta \theta_E)^2}$$

TABLE 2

DARK ENERGY (*XCDM* MODEL AND CPL PARAMETRIZATION) CONSTRAINTS OBTAINED ON THE FULL 118 STRONG LENSING (SL) SAMPLE.

Cosmology (Sample)	$w_0$	$w_1$	$\gamma_0$	$\gamma_1$
XCDM1 (SL; $\sigma_{ap}$ )	$w_0 = -1.45^{+0.54}_{-0.95}$	$w_1 = 0$	$\gamma_0 = 2.03 \pm 0.06$	$\gamma_1 = 0$
XCDM1 (SL; $\sigma_0$ )	$w_0 = -1.15^{+0.56}_{-1.20}$	$w_1 = 0$	$\gamma_0 = 2.07 \pm 0.07$	$\gamma_1 = 0$
XCDM2 (SL; $\sigma_{ap}$ )	$w_0 = -1.48^{+0.54}_{-0.94}$	$w_1 = 0$	$\gamma_0 = 2.06 \pm 0.09$	$\gamma_1 = -0.09 \pm 0.16$
XCDM2 (SL; $\sigma_0$ )	$w_0 = -1.35^{+0.67}_{-1.59}$	$w_1 = 0$	$\gamma_0 = 2.13^{+0.07}_{-0.12}$	$\gamma_1 = -0.09 \pm 0.17$
CPL1 (SL; $\sigma_{ap}$ )	$w_0 = -0.15^{+1.27}_{-1.60}$	$w_1 = -6.95^{+7.25}_{-3.05}$	$\gamma_0 = 2.08 \pm 0.09$	$\gamma_1 = -0.09 \pm 0.17$
CPL1 (SL; $\sigma_0$ )	$w_0 = -1.00^{+1.54}_{-1.95}$	$w_1 = -1.85^{+4.85}_{-6.75}$	$\gamma_0 = 2.14^{+0.07}_{-0.10}$	$\gamma_1 = -0.10 \pm 0.18$
CPL2 (SL; $\sigma_{ap}$ )	$w_0 = -0.10^{+1.21}_{-1.48}$	$w_1 = -0.25^{+6.25}_{-3.75}$	$\gamma_0 = 2.08$	$\gamma_1 = -0.09$
CPL2 (SL; $\sigma_0$ )	$w_0 = -1.05^{+1.43}_{-1.77}$	$w_1 = -1.65^{+4.25}_{-6.35}$	$\gamma_0 = 2.14$	$\gamma_1 = -0.10$
CPL2 (SN)	$w_0 = -1.00 \pm 0.40$	$w_1 = -0.12^{+1.58}_{-2.78}$	$\square$	$\square$

<sup>a</sup>In our fits we separately considered observed velocity dispersions  $\sigma_{ap}$  and corrected velocity dispersions  $\sigma_0$ , *XCDM1* corresponds to assumption of non-evolving power-law index  $\gamma$ , while *XCDM2* assumes its evolution  $\gamma(z) = \gamma_0 + \gamma_1 z_l$ . Fixed prior of  $\Omega_m = 0.315$  was assumed according to the Planck data. While fitting CPL parameters we assumed evolving lens mass density with  $\gamma_0$  and  $\gamma_1$  as free parameters (CPL1) and then fixed them at best-fit values (CPL2). For comparison fits of CPL parameters using Union2.1 supernovae data (SN) is shown.

# Strong lenses as standard(izable) rulers

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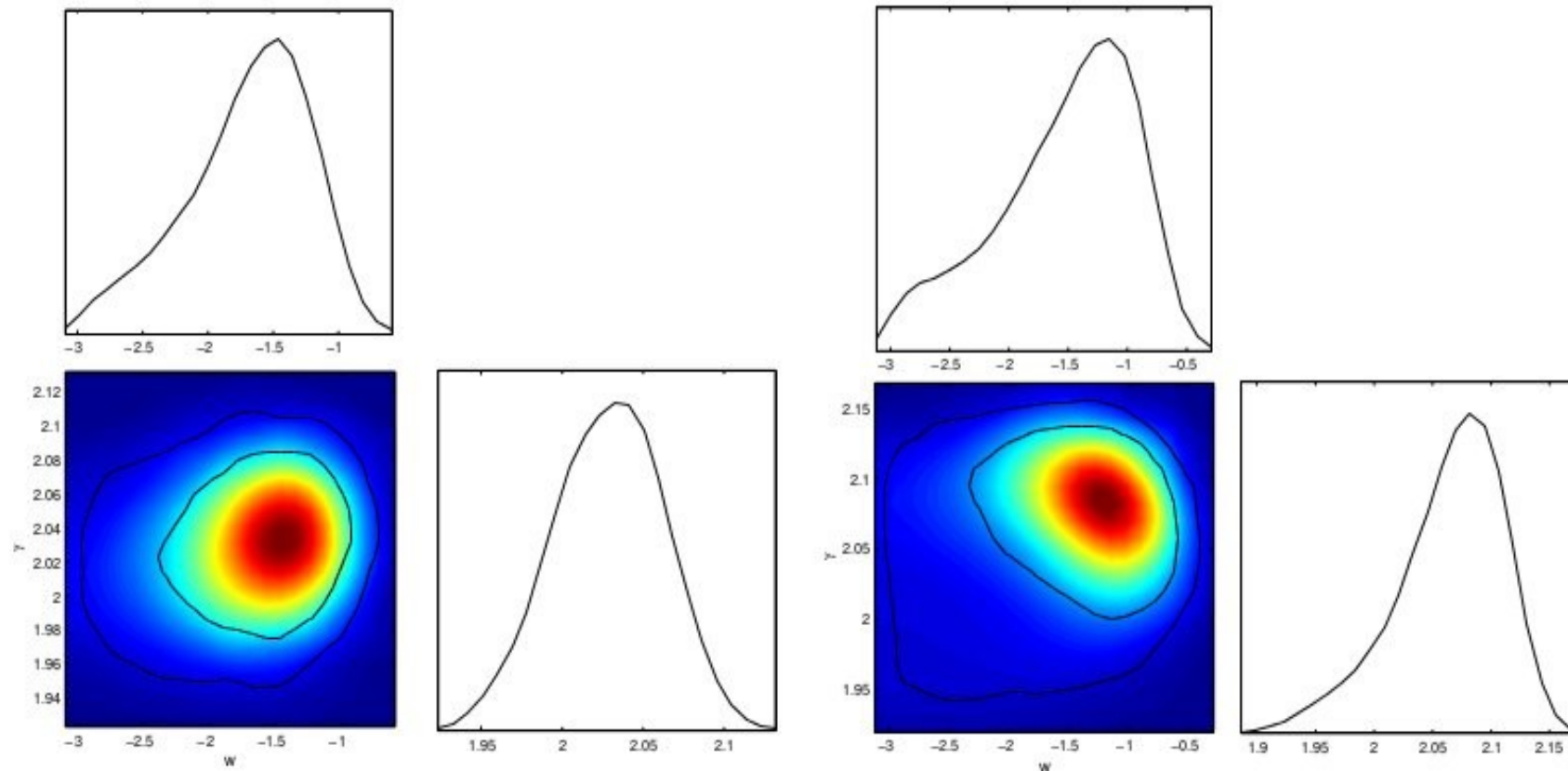


Fig. 3.— Joint fits of mass density slope  $\gamma$  and  $w$  coefficient in the XCDM model. Left panel shows the results obtained with velocity dispersion within the aperture  $\sigma_{ap}$  while on the right panel corrected velocity dispersion  $\sigma_0$  was used.  $\Omega_m = 0.315$  is assumed based on the Planck observations (Ade et al. 2014). Marginalized probability density functions for  $\gamma$  and  $w$  are also shown.

# Strong lenses as standard(izable) rulers

- **Third step forward:** taking into account possible evolution of the power-law Index  $\gamma$  with redshift –  $\gamma(z)$

mass density power-law index  $\gamma$  of massive ETG evolves with redshift !

[Ruff et al. 2011]  
[Brownstein et al. 2012]  
[Sonnenfeld et al. 2013]

$$\gamma(z_l) = 2.12_{-0.04}^{+0.03} - 0.25_{-0.12}^{+0.10} \times z_l + 0.17_{-0.02}^{+0.02}(\text{scatter})$$

therefore we also performed fit assuming linear relation:

$$\gamma(z_l) = \gamma_0 + \gamma_1 z_l$$

TABLE 2

DARK ENERGY (*XCDM* MODEL AND CPL PARAMETRIZATION) CONSTRAINTS OBTAINED ON THE FULL 118 STRONG LENSING (SL) SAMPLE.

Cosmology (Sample)	$w_0$	$w_1$	$\gamma_0$	$\gamma_1$
XCDM1 (SL; $\sigma_{ap}$ )	$w_0 = -1.45_{-0.95}^{+0.54}$	$w_1 = 0$	$\gamma_0 = 2.03 \pm 0.06$	$\gamma_1 = 0$
XCDM1 (SL; $\sigma_0$ )	$w_0 = -1.15_{-1.20}^{+0.56}$	$w_1 = 0$	$\gamma_0 = 2.07 \pm 0.07$	$\gamma_1 = 0$
XCDM2 (SL; $\sigma_{ap}$ )	$w_0 = -1.48_{-0.94}^{+0.54}$	$w_1 = 0$	$\gamma_0 = 2.06 \pm 0.09$	$\gamma_1 = -0.09 \pm 0.16$
XCDM2 (SL; $\sigma_0$ )	$w_0 = -1.35_{-1.50}^{+0.67}$	$w_1 = 0$	$\gamma_0 = 2.13_{-0.12}^{+0.07}$	$\gamma_1 = -0.09 \pm 0.17$
CPL1 (SL; $\sigma_{ap}$ )	$w_0 = -0.15_{-1.60}^{+1.27}$	$w_1 = -0.95_{-3.05}^{+7.25}$	$\gamma_0 = 2.08 \pm 0.09$	$\gamma_1 = -0.09 \pm 0.17$
CPL1 (SL; $\sigma_0$ )	$w_0 = -1.00_{-1.93}^{+1.54}$	$w_1 = -1.85_{-6.73}^{+4.85}$	$\gamma_0 = 2.14_{-0.10}^{+0.07}$	$\gamma_1 = -0.10 \pm 0.18$
CPL2 (SL; $\sigma_{ap}$ )	$w_0 = -0.16_{-1.48}^{+1.21}$	$w_1 = -6.25_{-3.75}^{+6.25}$	$\gamma_0 = 2.08$	$\gamma_1 = -0.09$
CPL2 (SL; $\sigma_0$ )	$w_0 = -1.05_{-1.77}^{+1.43}$	$w_1 = -1.65_{-6.35}^{+4.25}$	$\gamma_0 = 2.14$	$\gamma_1 = -0.10$
CPL2 (SN)	$w_0 = -1.00 \pm 0.40$	$w_1 = -0.12_{-2.78}^{+1.58}$	□	□

<sup>a</sup>In our fits we separately considered observed velocity dispersions  $\sigma_{ap}$  and corrected velocity dispersions  $\sigma_0$ , *XCDM1* corresponds to assumption of non-evolving power-law index  $\gamma$ , while *XCDM2* assumes its evolution  $\gamma(z) = \gamma_0 + \gamma_1 z_l$ . Fixed prior of  $\Omega_m = 0.315$  was assumed according to the Planck data. While fitting CPL parameters we assumed evolving lens mass density with  $\gamma_0$  and  $\gamma_1$  as free parameters (CPL1) and then fixed them at best-fit values (CPL2). For comparison fits of CPL parameters using Union2.1 supernovae data (SN) is shown.

# Strong lenses as standard(izable) rulers

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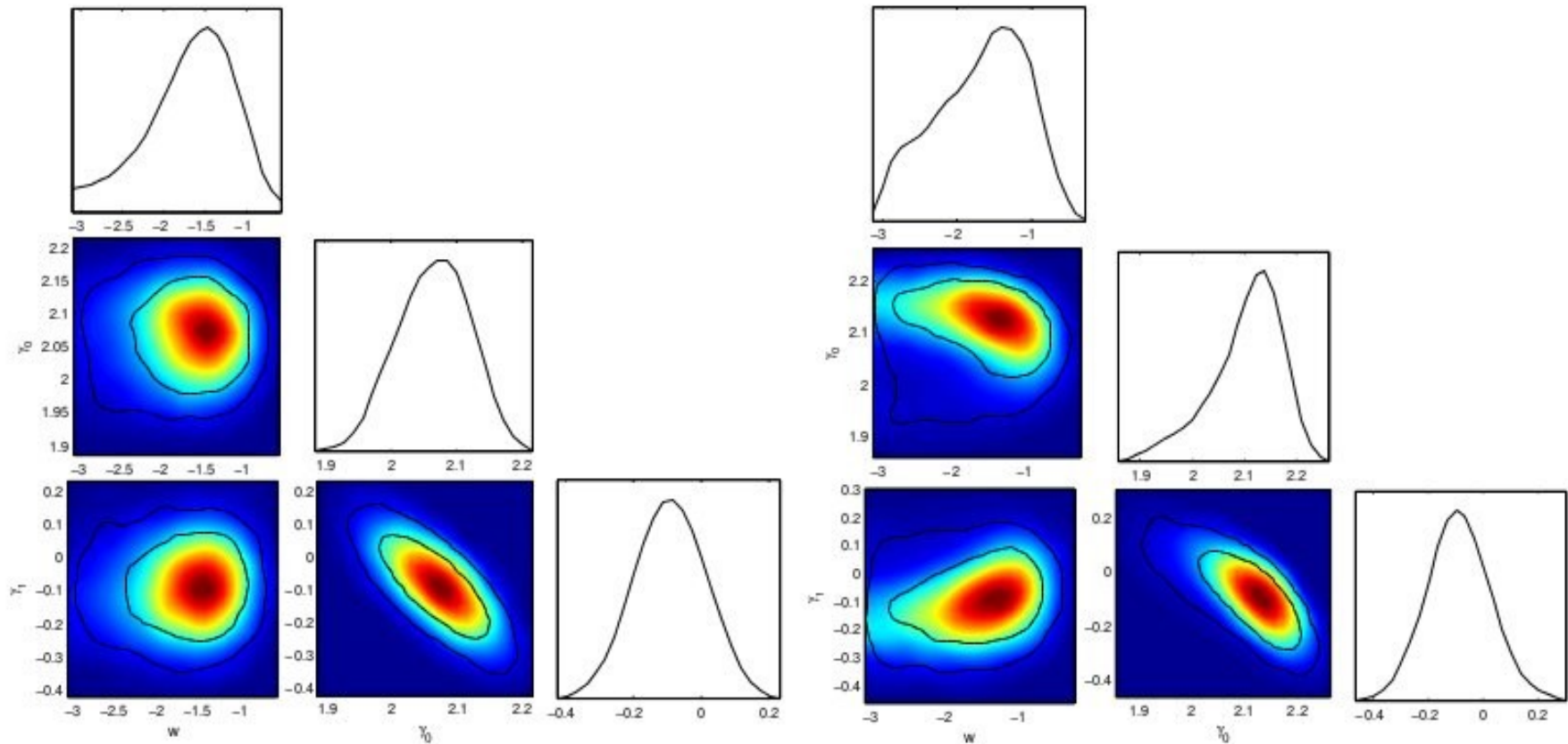


Fig. 4.— Joint fits of  $w$  coefficient in the XCDM model and mass density slope parameters  $(\gamma_0, \gamma_1)$  in evolving slope scenario  $\gamma(z) = \gamma_0 + \gamma_1 z_l$ . Left and right panels display the results obtained by using  $\sigma_{ap}$  and  $\sigma_0$  respectively.

# Strong lenses as standard(izable) rulers

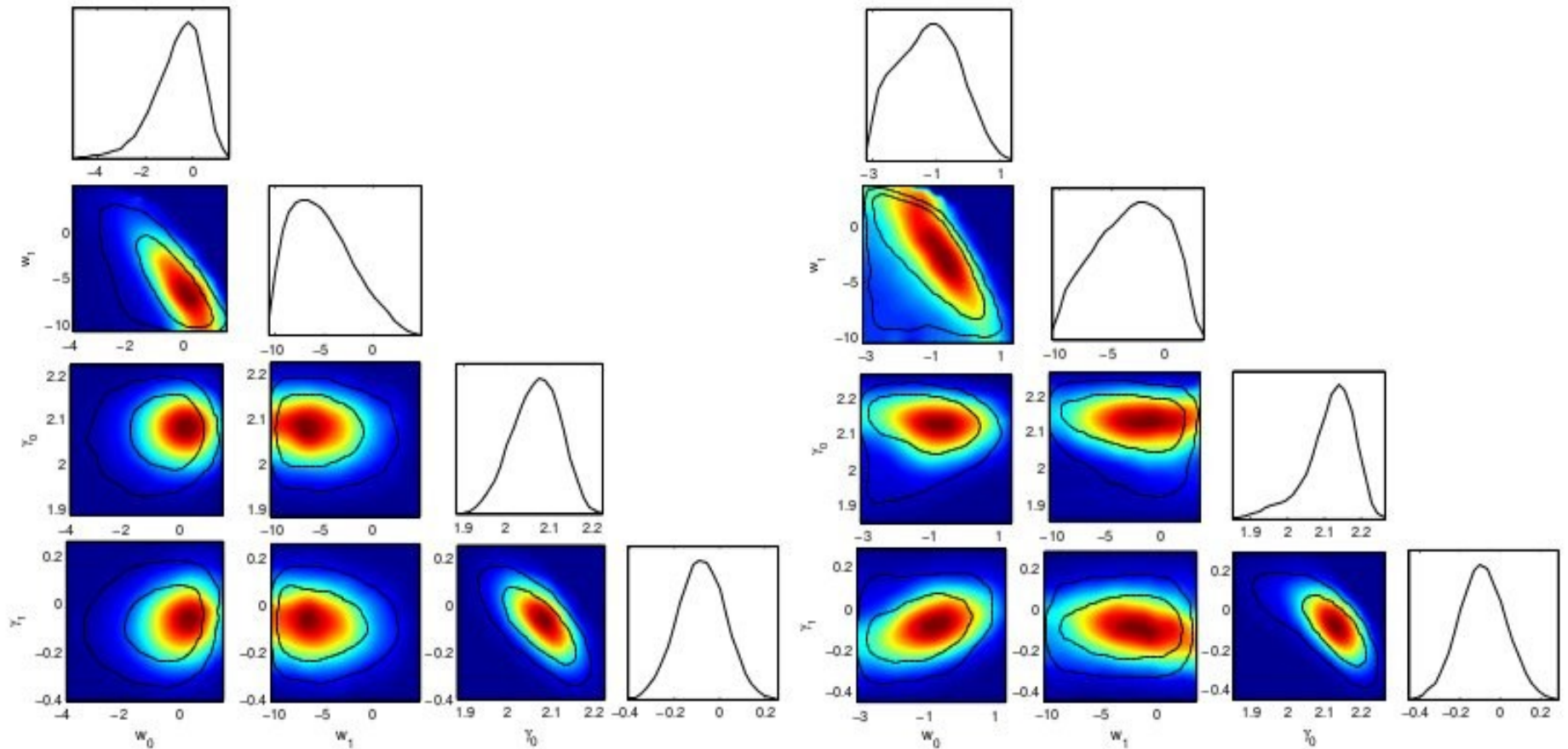


Fig. 5.— Joint fits of  $(w_0, w_1)$  evolving cosmic equation of state coefficients in the CPL parametrization. Lensing galaxies were assumed to have evolving mass density slope  $\gamma(z) = \gamma_0 + \gamma_1 z$  and  $(\gamma_0, \gamma_1)$  parameters. Left and right panels display the results obtained by using  $\sigma_{ap}$  and  $\sigma_0$  respectively.

# Strong lenses as standard(izable) rulers

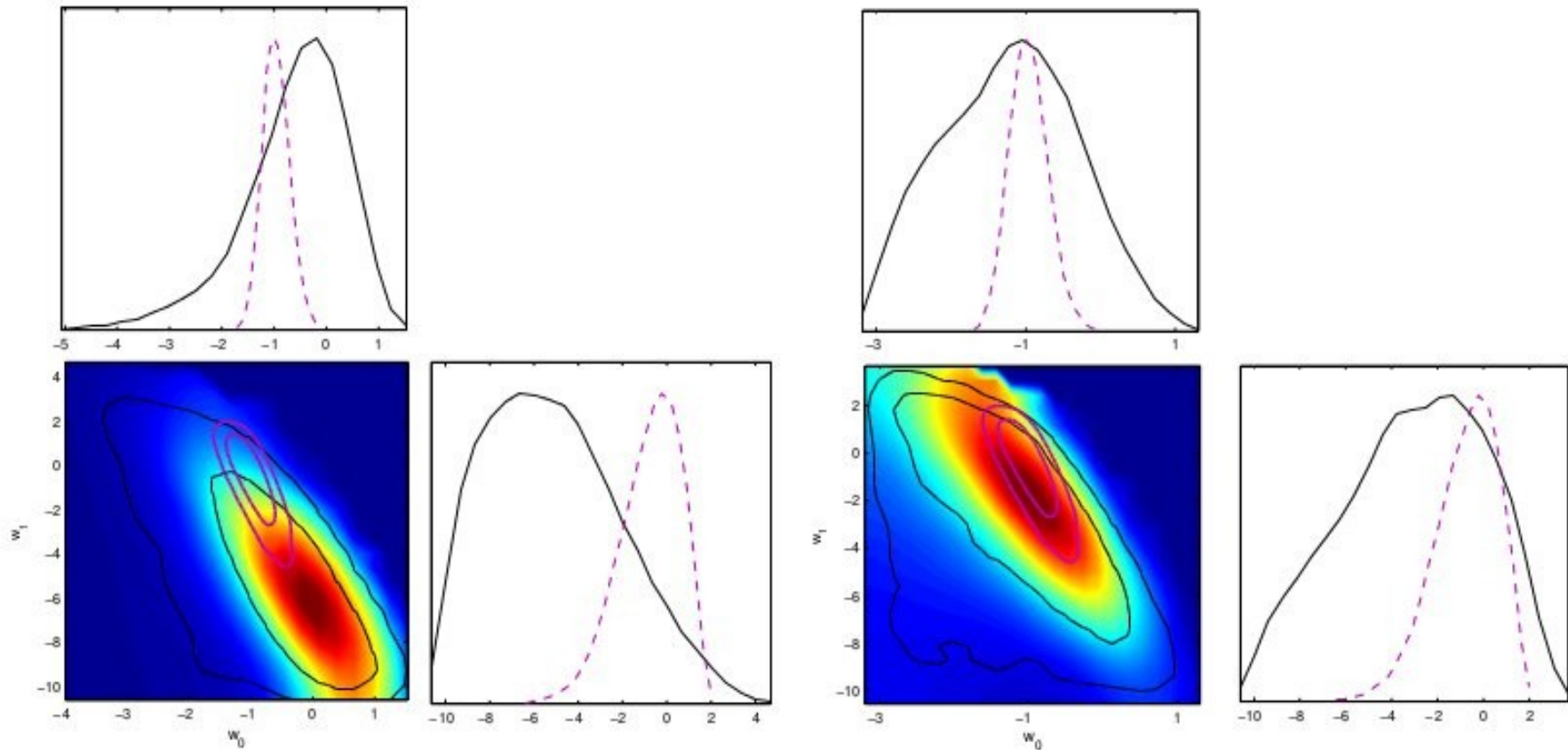


Fig. 6.— Joint fits of  $(w_0, w_1)$  evolving cosmic equation of state coefficients in the CPL parametrization. Lensing galaxies were assumed to have evolving mass density slope  $\gamma(z) = \gamma_0 + \gamma_1 z_l$  and  $(\gamma_0, \gamma_1)$  parameters were fixed at our best-fit values in  $\Lambda$ CDM model. Left and right panels display the results obtained by using  $\sigma_{ap}$  and  $\sigma_0$  respectively.

# Strong lenses as standard(izable) rulers

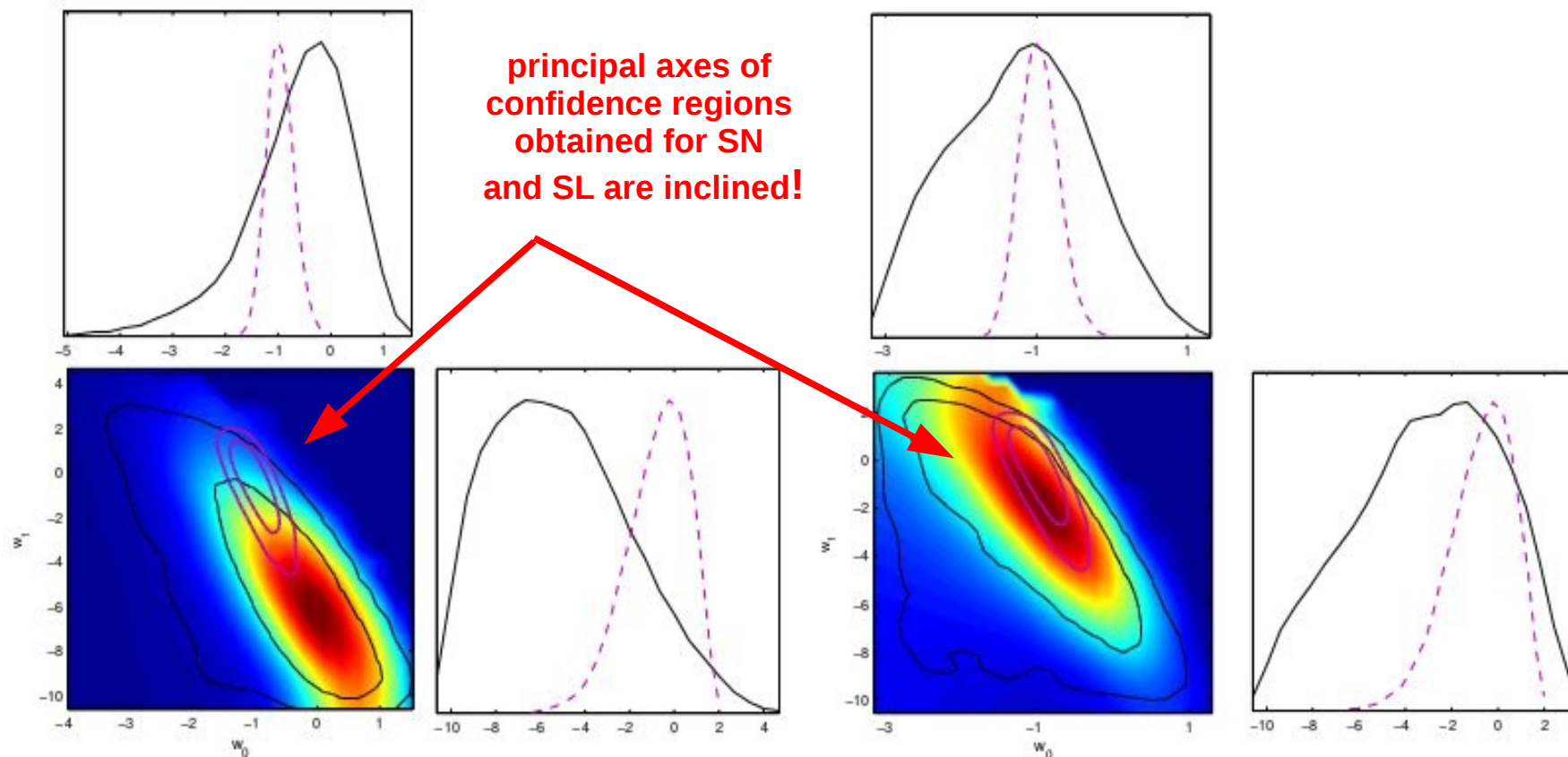


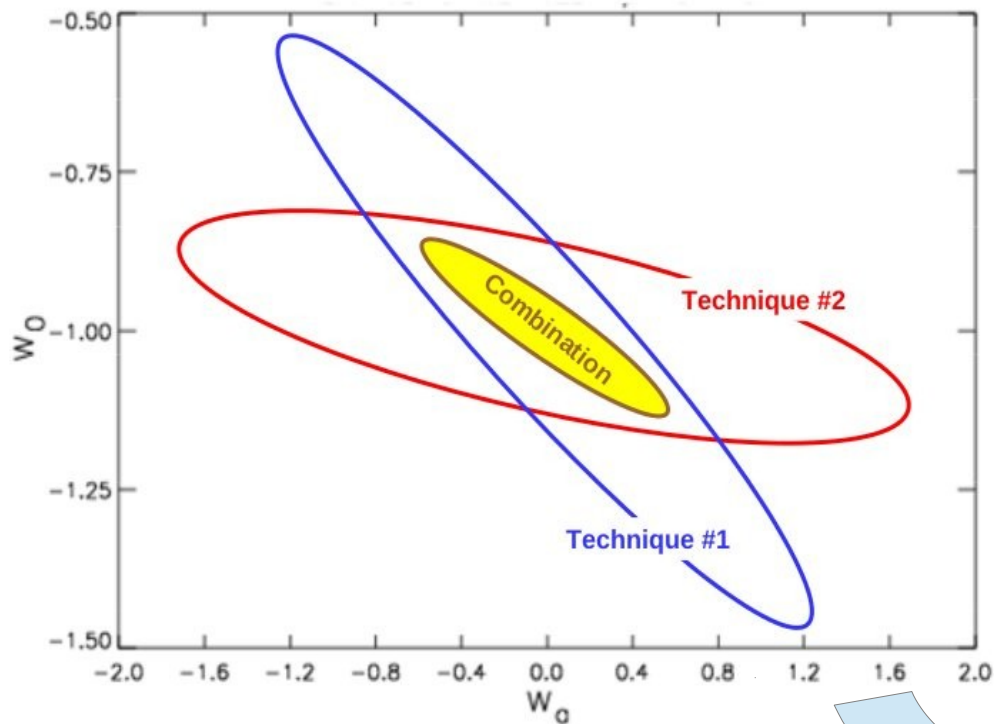
Fig. 6.— Joint fits of  $(w_0, w_1)$  evolving cosmic equation of state coefficients in the CPL parametrization. Lensing galaxies were assumed to have evolving mass density slope  $\gamma(z) = \gamma_0 + \gamma_1 z_l$  and  $(\gamma_0, \gamma_1)$  parameters were fixed at our best-fit values in  $\Lambda$ CDM model. Left and right panels display the results obtained by using  $\sigma_{ap}$  and  $\sigma_0$  respectively.

this inclination is higher than in previous studies

# Dark Energy Complementarity

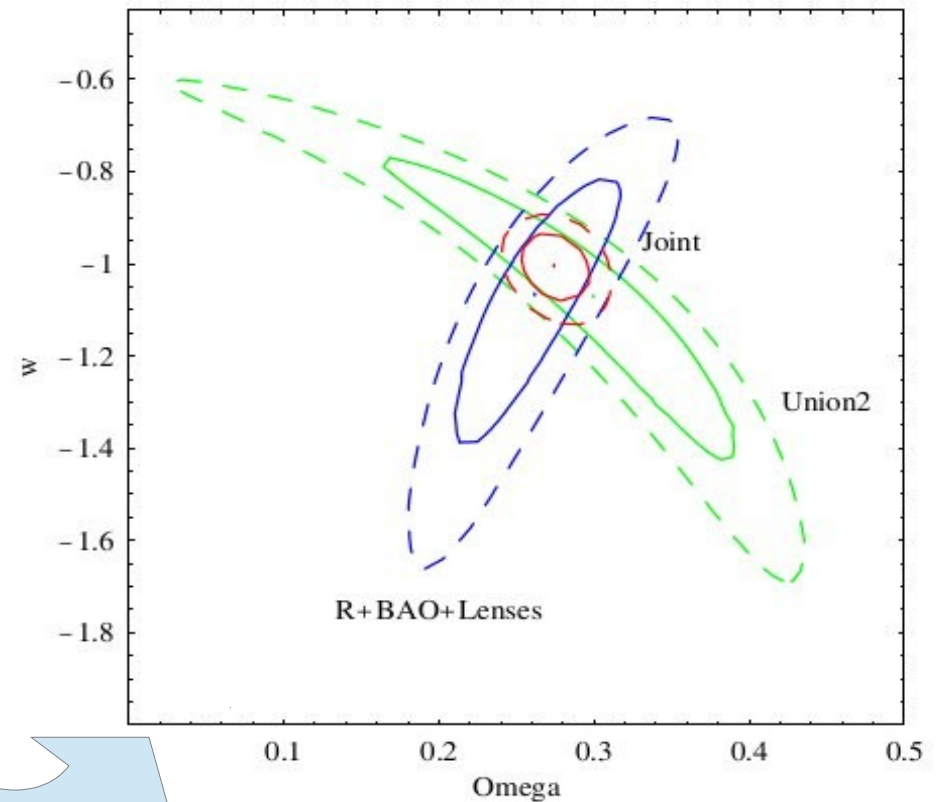
- We expect that the greatest accuracy and confidence in the measurements will come from independent crosschecks and complementarity between different methods probing the cosmology:

Albrecht et al. (DETF) 2006



*Illustration of the power of combining techniques.*

Biesiada, Malec, AP, 2011



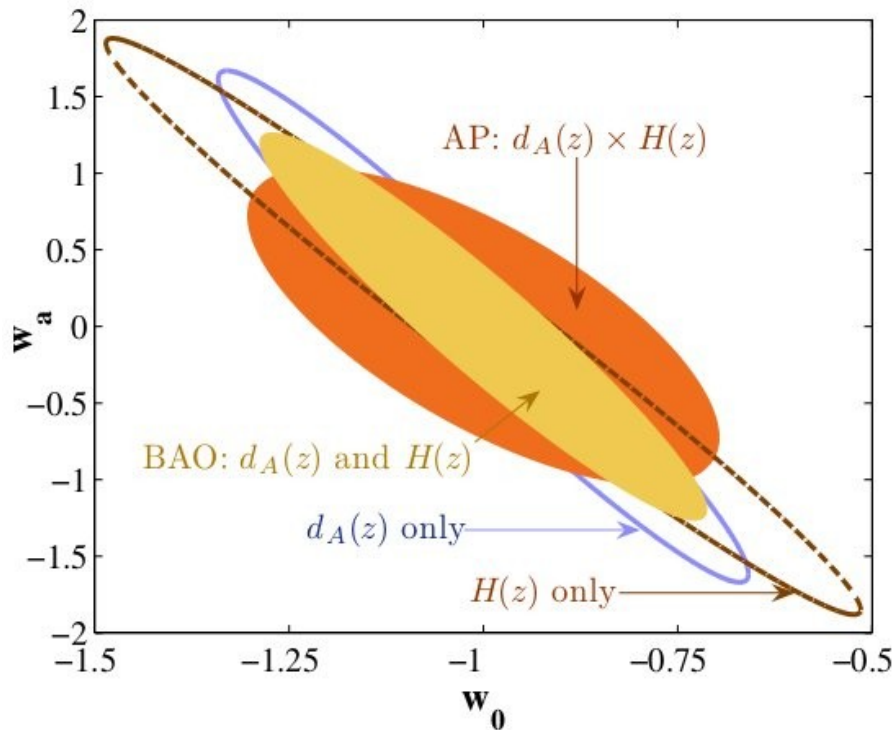
just like complementarity of standard rulers and standard candles in  $\Omega$ - $w$  parameter plane



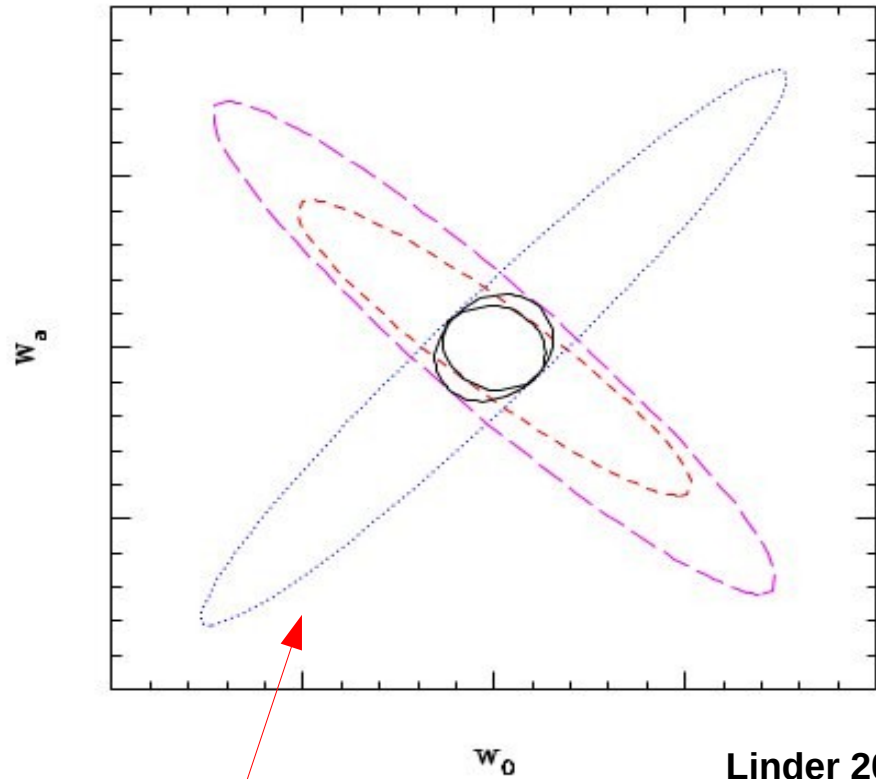
# Dark Energy Complementarity

- Problem: all the known methods of distance measurements possess a similar fundamental dependence on the cosmic equation of state through the Hubble parameter, or expansion rate.

Complementarity between methods can only be partial in  $w_0$ - $w_a$  parameter plane !



Bassett, Hlozek et al. 2009

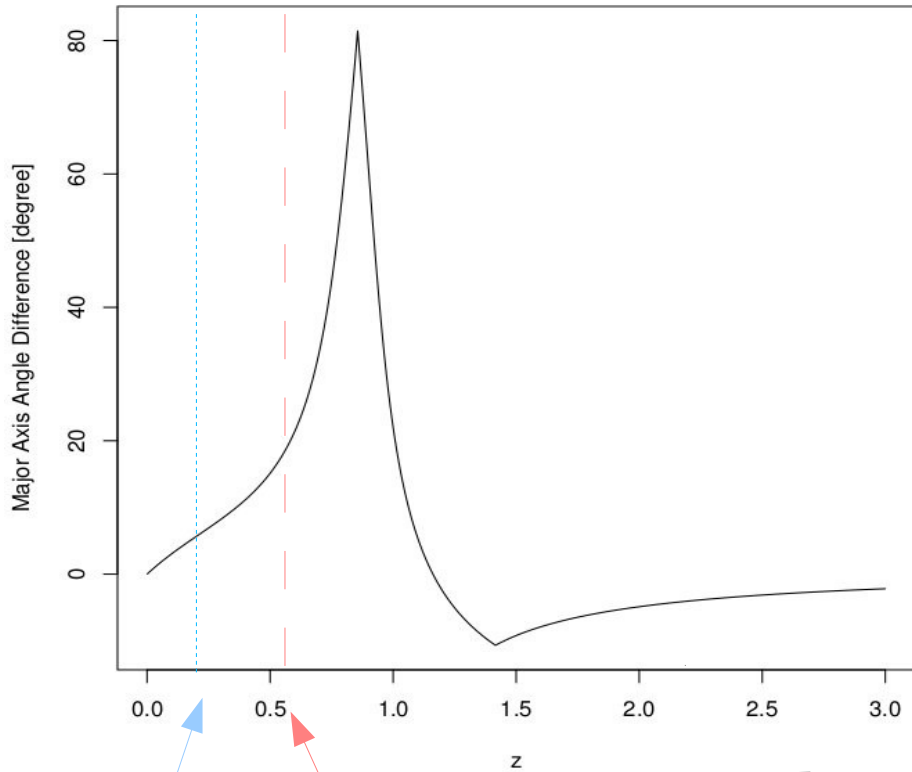


Linder 2004

**Breaking degeneracy: construction of a cosmological probe whose sensitivity lies orthogonally in the  $w_0$ - $w_1$  parameter plane**

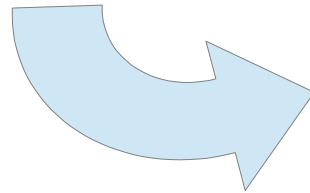
# Complementarity of strong lensing measurements

- Strong lensing measurements are not perfect orthogonal to other distance measurement methods in the  $w_0$ - $w_a$  plane but to a certain extent they can be considered as complementary:

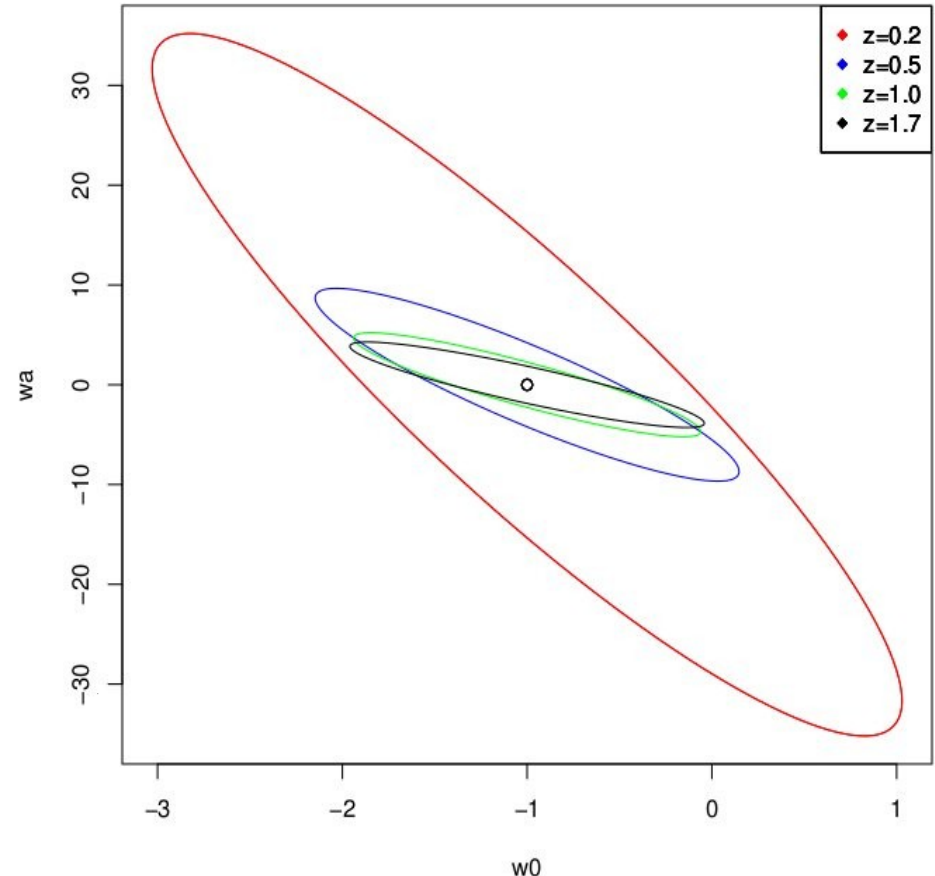


SLACS sample

BELLS sample



confidence contours for an idealized experiment measuring the distance ratio for several samples with different redshifts:

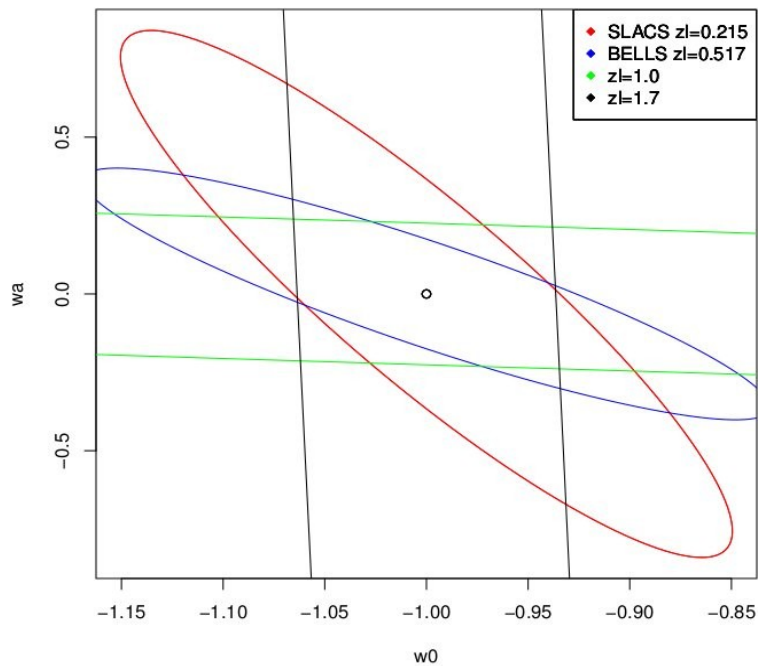


major axis angle of confidence contour for an idealized experiment as a function of redshift:

# Complementarity of strong lensing measurements

## Fisher matrix analysis:

- The greatest accuracy and confidence in the measurements of dark energy parameters can be achieved by independent crosschecks and complementarity between different observations
- Strong lensing measurements may help to break  $w_0$ - $w_a$  degeneracy - the angle of the major axis of the confidence contour depends on the redshift of the sample

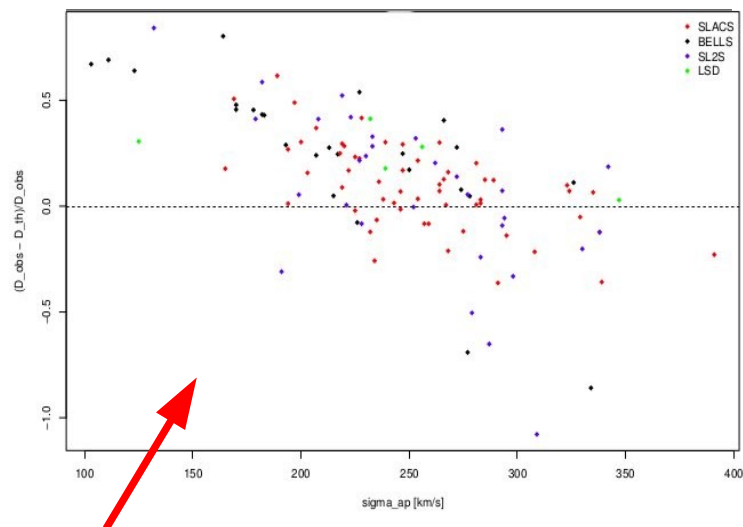
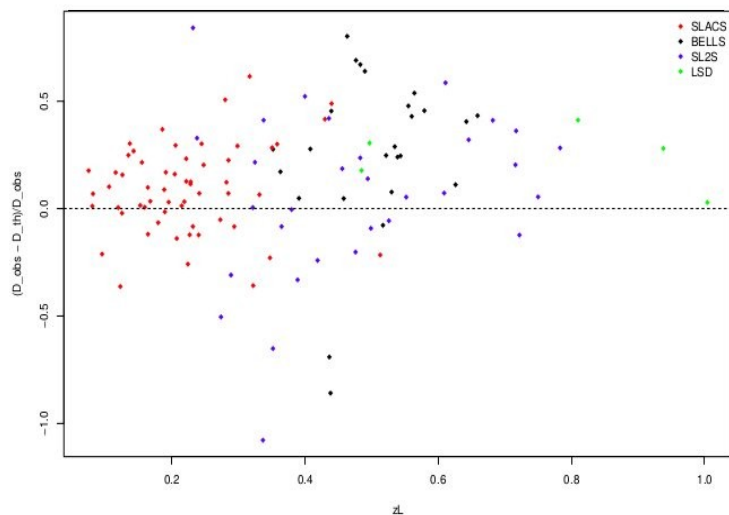
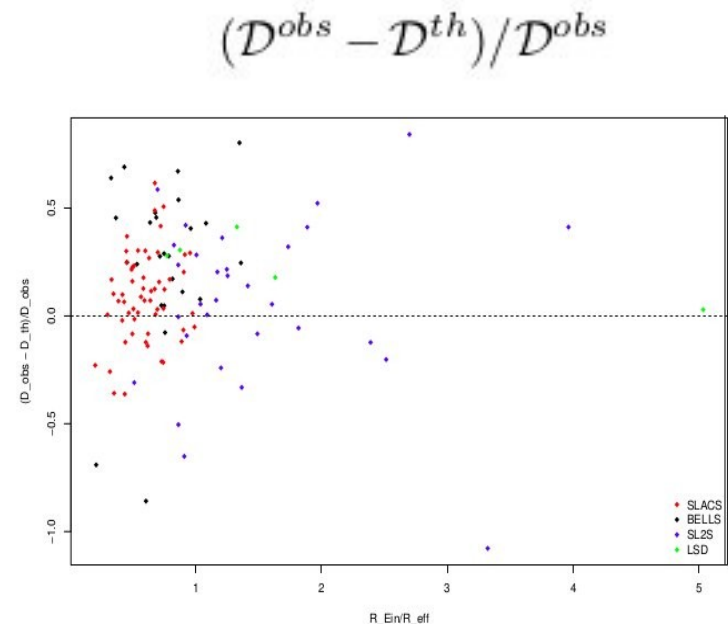
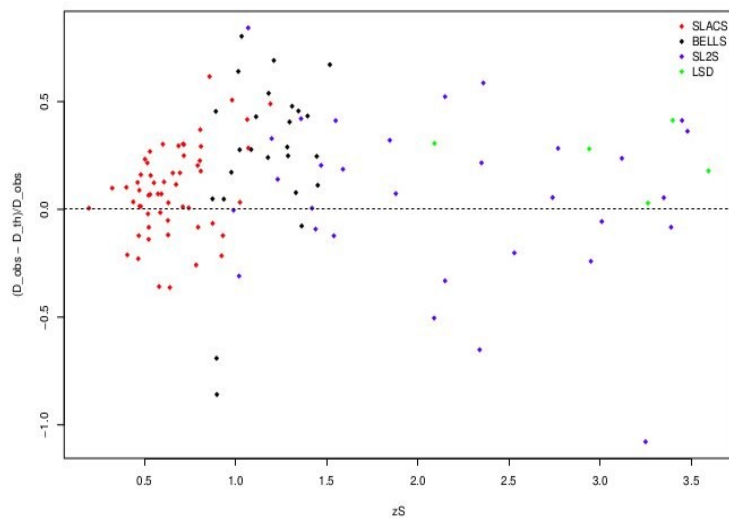


**offer some complementarity in  $w_0$ - $w_a$  parameters plane**

**confidence contours for an idealized experiment measuring the distance ratio for several samples with different redshifts:**

# Strong lenses as standard(izable) rulers

- Fourth step forward: study of systematics  
– diagnostics of residuals



# Strong lenses as standard(izable) rulers

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- Fourth step forward: study of systematics – further discussion.

(correlation coeff. ca. -0.6)

- Noticeable anti-correlation visible in residuals as a function of the velocity dispersion is especially pronounced for lenses with  $\sigma_0 < 230$  km/s. If one excluded small velocity dispersion lenses the result would be comparable to other diagnostics discussed.

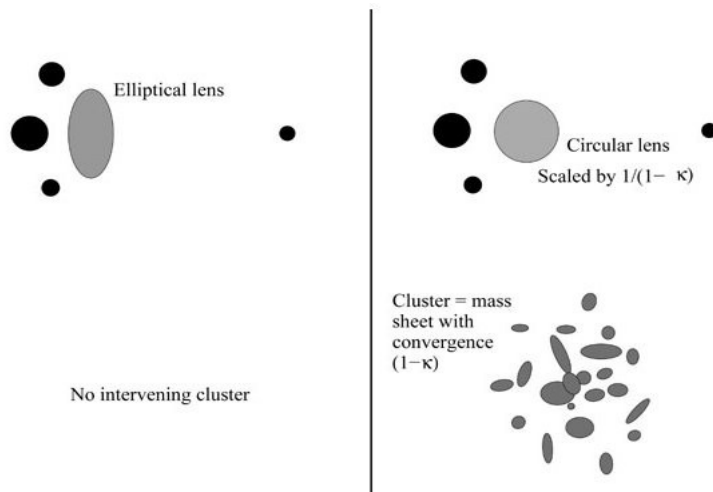
elliptical galaxies with smaller observed velocity dispersions are more centrally concentrated

[Shu et al. 2014]

(scatter at the level of +/-50%)

- Individual properties of lenses (environment, deviation from spherical symmetry) are crucial for our method (not considered here)
- „mass-sheet degeneracy” problem

contamination of secondary lenses – clumps of matter along or near the line of sight



[Falco, Gornstein, Shapiro, 1985]

[Schneider&Sluse, 2013]

after Courbin

[<http://ned.ipac.caltech.edu/level5/March03/Courbin/>]

# Conclusions and prospects:

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- Our analysis demonstrates that strong gravitationally lensed systems can already now be used to probe cosmological parameters, especially the cosmic equation of state for dark energy.
- Further developments of this idea needed

sources of systematics:

[Koopmans et al. 2005, 2006]

- in our analysis power-law index  $\gamma$  should be understood as an effective descriptor capturing both the density profile and anisotropy parameter of velocity dispersions ( $\beta$ )

[Barnabe, Spiniello & Koopmans, 2014]

- three-dimensional shape of galaxies (prolatness/oblatness) [Chae et al. 2003]
- influence of the line-of-sight contamination

- Careful choice of the sample in terms of lens and source redshifts will allow for cosmological analysis in a complementary way

breaking degeneracy between dark energy parameters

[Piórkowska et al. 2011]

Thank you for your attention

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**[aleksandra.piorkowska@us.edu.pl](mailto:aleksandra.piorkowska@us.edu.pl)**