Vacuum stability in the Standard Model and its extensions

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based on:

Z. Lalak, P. Olszewski and ML, JHEP **1405**, 119 (2014) arXiv:1402.3826

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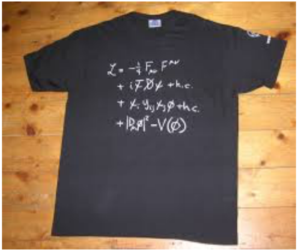
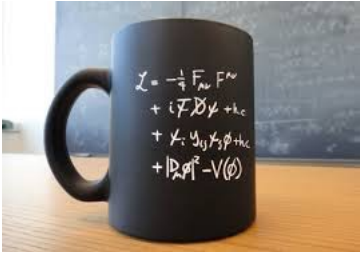
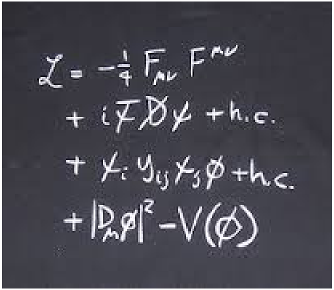
Standard Model







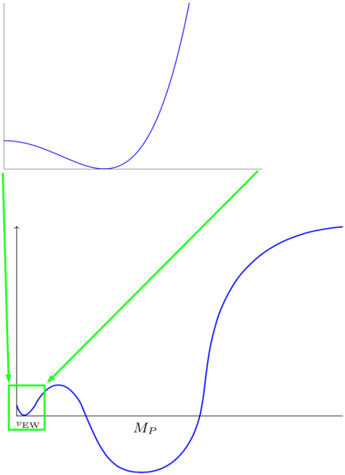
Scalar Potential

- Classically

$$V_{SM}^{tree} = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4$$

- Quantum corrections

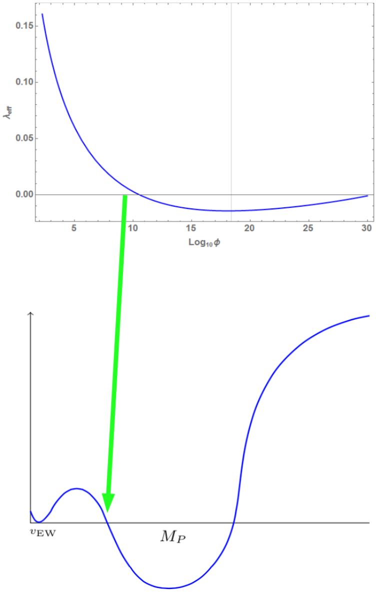
$$V_{SM}^{1-loop} = -\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 + \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i\right]$$



$$\Delta V_{SM}^{1-loop} = \sum_i \frac{n_i}{64\pi^2} M_i^4 \left[\ln\left(\frac{M_i^2}{\mu^2}\right) - C_i \right]$$

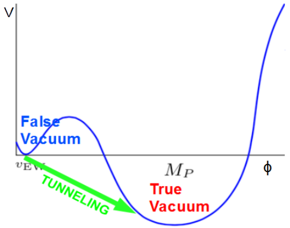
- For large field values
  $m^2 << \phi^2 \rightarrow M_i \propto \phi$
- Expansion under control if
  $\mu = \phi$

$$V_{SM}(\phi) \approx \frac{\lambda_{eff}(\phi)}{4} \phi^4$$



Tunneling

- Vacuum decay proceeds through nucleation of true vacuum bubbles within false vacuum.
  S. R. Coleman, Phys. Rev. D **15** (1977) 2929.
  C. G. Callan, Jr. and S. R. Coleman, Phys. Rev. D **16** (1977) 1762.

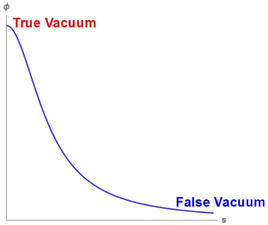


- Bubble: $O(4)$ symmetric solution of euclidean EOM:

$$\ddot{\phi} + \frac{3}{s}\dot{\phi} = \frac{\partial V(\phi)}{\partial \phi}, \quad s = \sqrt{\tau^2 + \vec{x}^2}.$$

with

  - $\dot{\phi}(s = 0) = 0$ at the true vacuum
  - $\phi(s = \infty) = \phi_{min}$ at the false vacuum



Tunneling

- Decay probability dp of a volume d^3x

$$dp = dt d^3x \frac{S_E^2}{4\pi^2} \left| \frac{det'[-\partial^2 + V''(\phi)]}{det[-\partial^2 + V''(\phi_0)]} \right|^{-1/2} e^{-S_E}.$$

- Action of the bounce solution

$$S_E = 2\pi^2 \int ds s^3 \left(\frac{1}{2}\dot{\phi}^2(s) + V(\phi(s)) \right).$$

- Simplifying:
  - prefactor replaced with width of the barrier $\propto \phi^4(s=0)$
  - volume of the universe approximated by $T_U^3 = (10^{10}\mathrm{yr})^3$

Expected lifetime of the false vacuum ($p(\tau) = 1$):

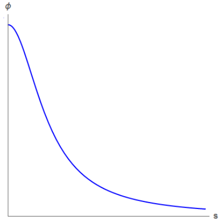
$$\frac{\tau}{T_U} = \frac{1}{\phi_0^4 T_U^4} e^{S_E}$$

Standard Model

- Anlytical solution for quartic potential (with $\lambda = \text{const} < 0$):

$$V(\phi) = \frac{\lambda}{4}\phi^4 \implies S_E = \frac{8\pi^2}{3|\lambda|}$$

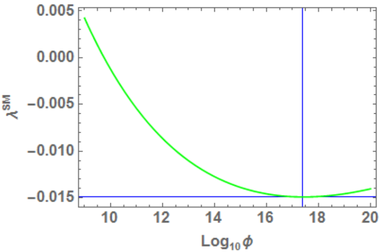
K. M. Lee and E. J. Weinberg, Nucl. Phys. B **267** (1986) 181.



- Approximating SM with a constant λ:

$$\frac{\tau}{T_U} = \frac{1}{\phi^4(\lambda_{min})\,T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}} \approx 10^{596},$$

we obtain a lower bound for ϕ that minimizes $\lambda_{eff}(\phi)$.

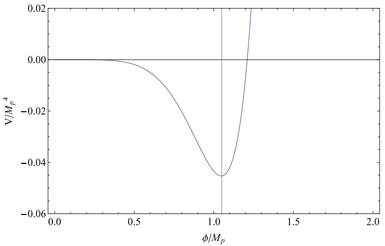


Effective potential with nonrenormalisable interactions

- Nonrenormalisable couplings modify the potential around the Planck scale:

$$V \approx \frac{\lambda_{eff}(\phi)}{4}\phi^4 + \frac{\lambda_6}{6!}\frac{\phi^6}{M_p^2} + \frac{\lambda_8}{8!}\frac{\phi^8}{M_p^4},$$

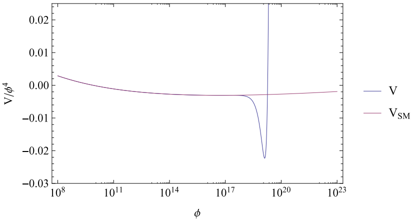
V. Branchina and E. Messina, Phys. Rev. Lett. **111** (2013) 241801.



- Simple quartic potential approximation:

$$\lambda_{eff}^{NEW}(\phi) = 4\frac{V}{\phi^4} =$$

$$\lambda_{eff}^{SM}(\phi) + 4\frac{\lambda_6}{6!}\frac{\phi^2}{M_p^2} + 4\frac{\lambda_8}{8!}\frac{\phi^4}{M_p^4}.$$



Numerical vs Analytical

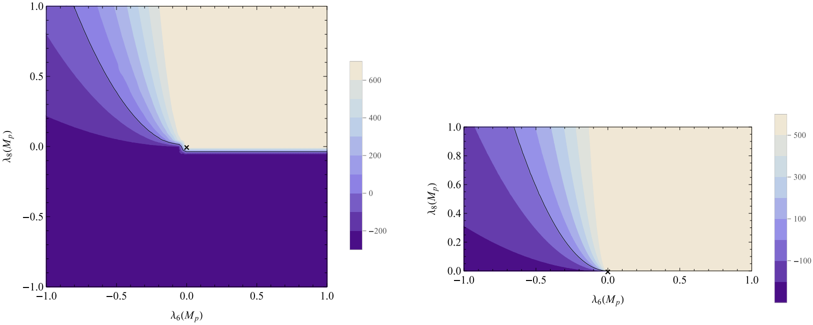


Figure: $\mathrm{Log}_{10}(\frac{\tau}{T_u})$ calculated numerically (left panel) and analytically (right panel).

RG improvement

- Small correction to SM parameters

$$\Delta\beta_\lambda = \frac{\lambda_6}{16\pi^2}\frac{m^2}{M_p^2}.$$

- One-loop beta functions of new couplings

$$\begin{aligned} 16\pi^2\beta_{\lambda_6} &= \frac{10}{7}\lambda_8\frac{m^2}{M^2} + 18\lambda_6 6\lambda - 6\lambda_6\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right), \\ 16\pi^2\beta_{\lambda_8} &= \frac{7}{5}28\lambda_6^2 + 30\lambda_8 6\lambda - 8\lambda_8\left(\frac{9}{4}g_2^2 + \frac{9}{20}g_1^2 - 3y_t^2\right), \end{aligned}$$

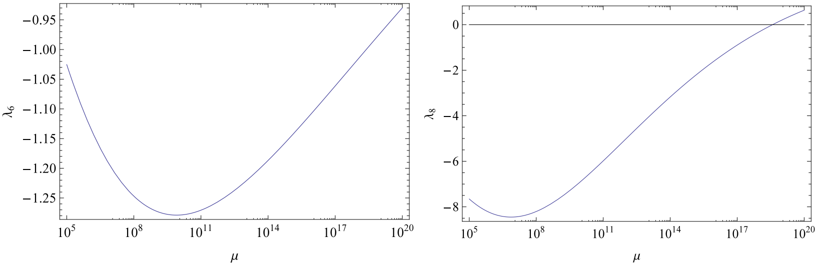


Figure: Example solution with $\lambda_6(M_p) = -1$ and $\lambda_8(M_p) = -0.1$.

Numerical vs Analytical with RG improvement

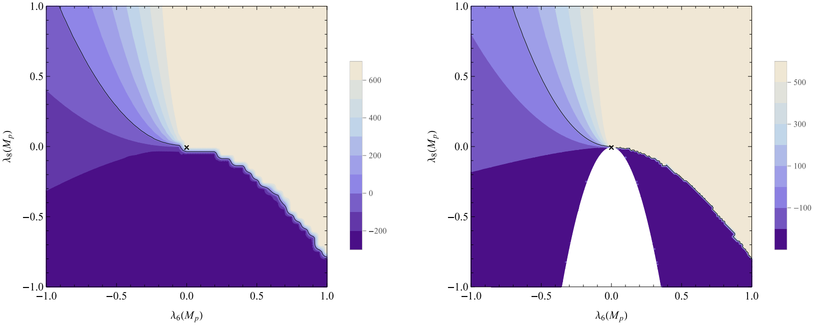


Figure: $\mathrm{Log}_{10}(\frac{\tau}{T_u})$ calculated numerically (left panel) and analytically (right panel).

Comparison

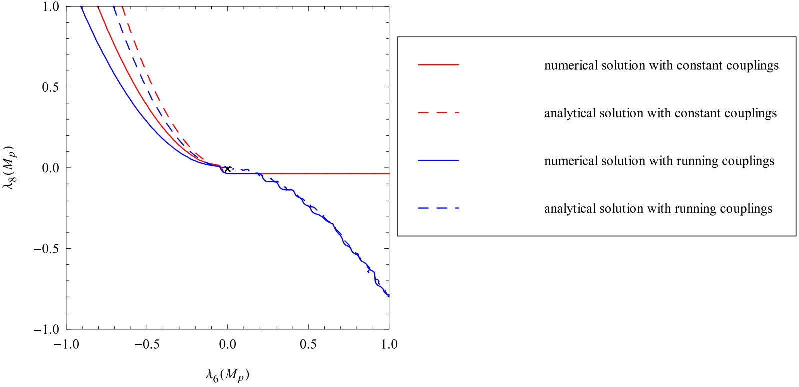
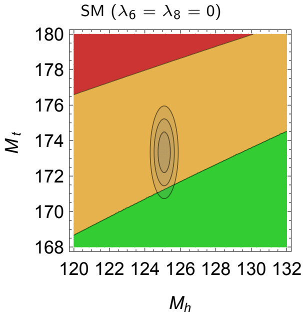
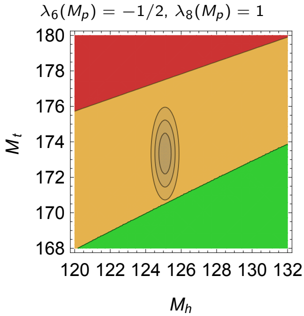
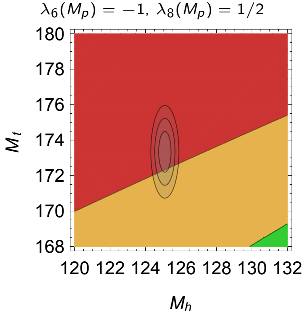


Figure: Metastability boundary ($\tau = T_u$) obtained using different methods.

SM phase diagram







Magnitude of the suppression scale

Approximate lifetime:

$$\frac{\tau}{T_U} = \frac{1}{\mu^4(\lambda_{min})\,T_U^4} e^{\frac{8\pi^2}{3|\lambda_{min}|}}.$$

Positive λ_6 and λ_8 $\rightarrow$ stabilizing the potential

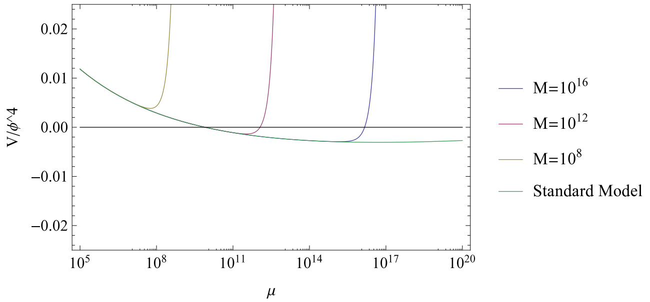


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = \lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$ are, respectively, $\log_{10}(\frac{\tau}{T_U}) = \infty, 1302, 581$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Magnitude of the suppression scale

Positive λ_8 and negative λ_6 $\rightarrow$ New Minimum

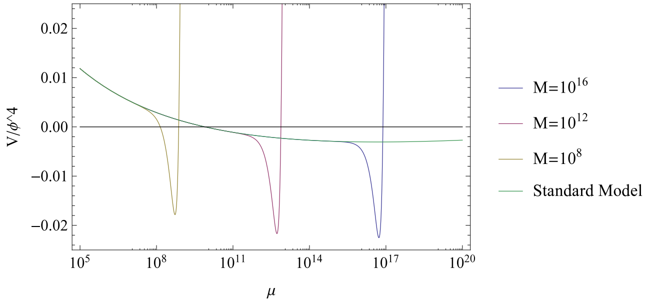


Figure: Scale dependence of $\frac{\lambda_{eff}}{4} = \frac{V}{\phi^4}$ with $\lambda_6 = -1$ and $\lambda_8 = 1$ for different values of suppression scale M. The lifetimes corresponding to suppression scales $M = 10^8, 10^{12}, 10^{16}$, are, respectively, $\log_{10}(\frac{\tau}{T_U}) = -45, -90, -110$ while for the Standard Model $\log_{10}(\frac{\tau}{T_U}) = 540$.

Conclusions

- Analytical approximation of vacuum lifetime is fairly accurate
- RG improvement stabilizes significant parts of the parameter space
- SM vacuum can be stabilized by new physics interactions only if they appear at low enough energy scale $\approx 10^{10} - 10^{11}$ GeV
- SM vacuum lifetime can be dramatically shortened by new physics at any scale