Uniwersytet Wrocławski

Quark Matter in Compact Stars Vector enhanced BAG model

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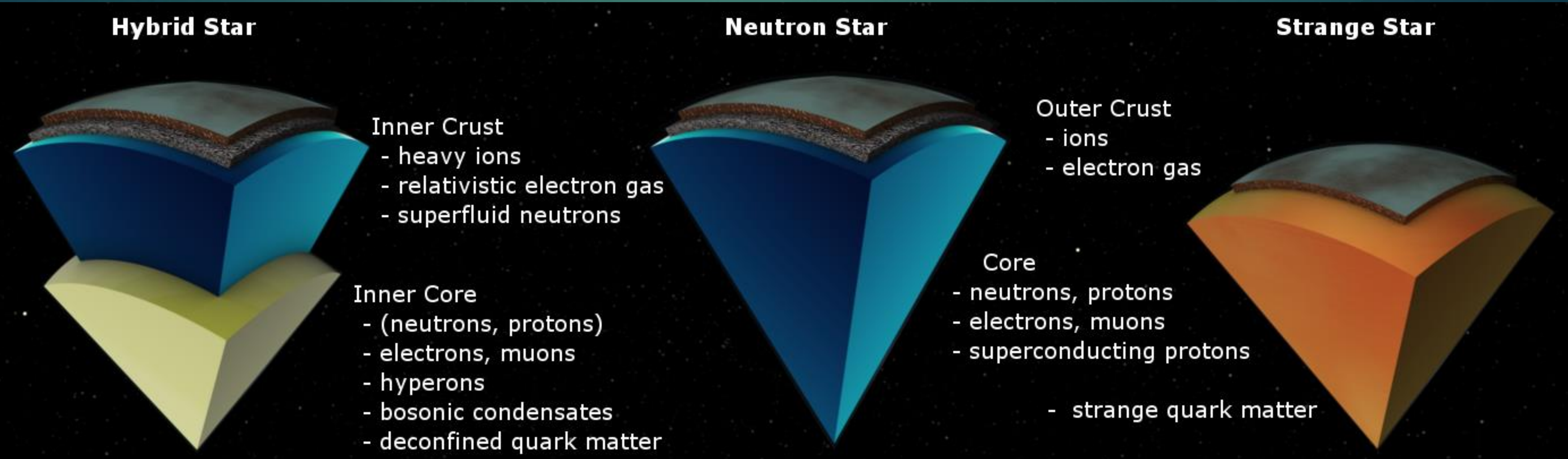
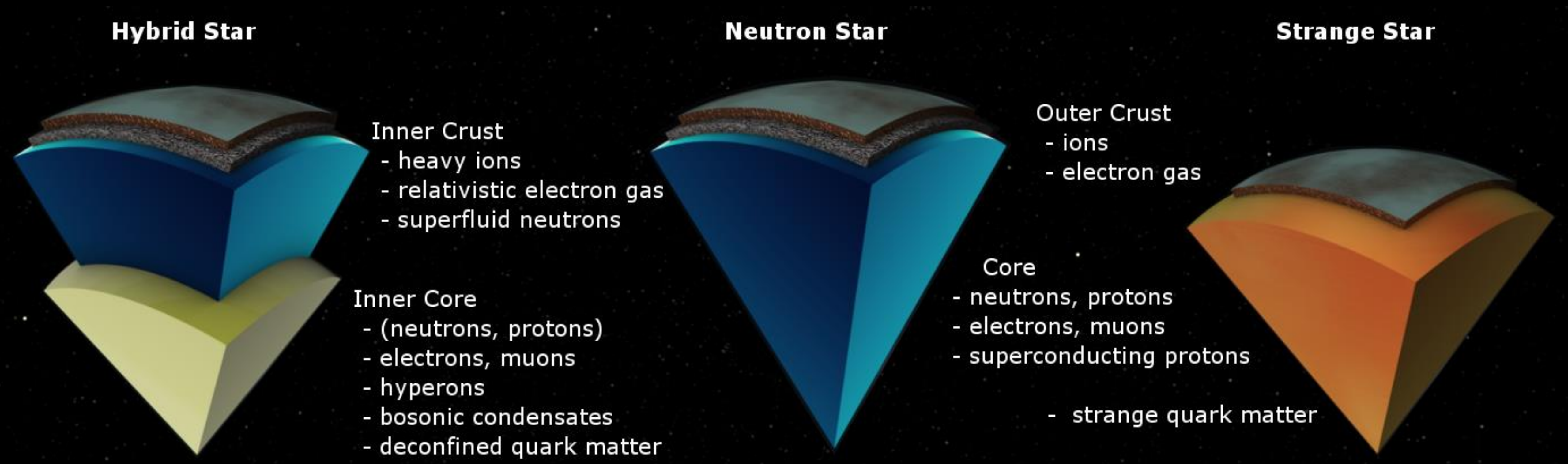
Neutron Stars

- Variety of scenarios regarding inner structure: with or without QM
- Question whether/how QCD phase transition occurs is not settled
- Most honest approach: take both (and more) scenarios into account and compare to available data



Neutron Stars = Quark Cores?



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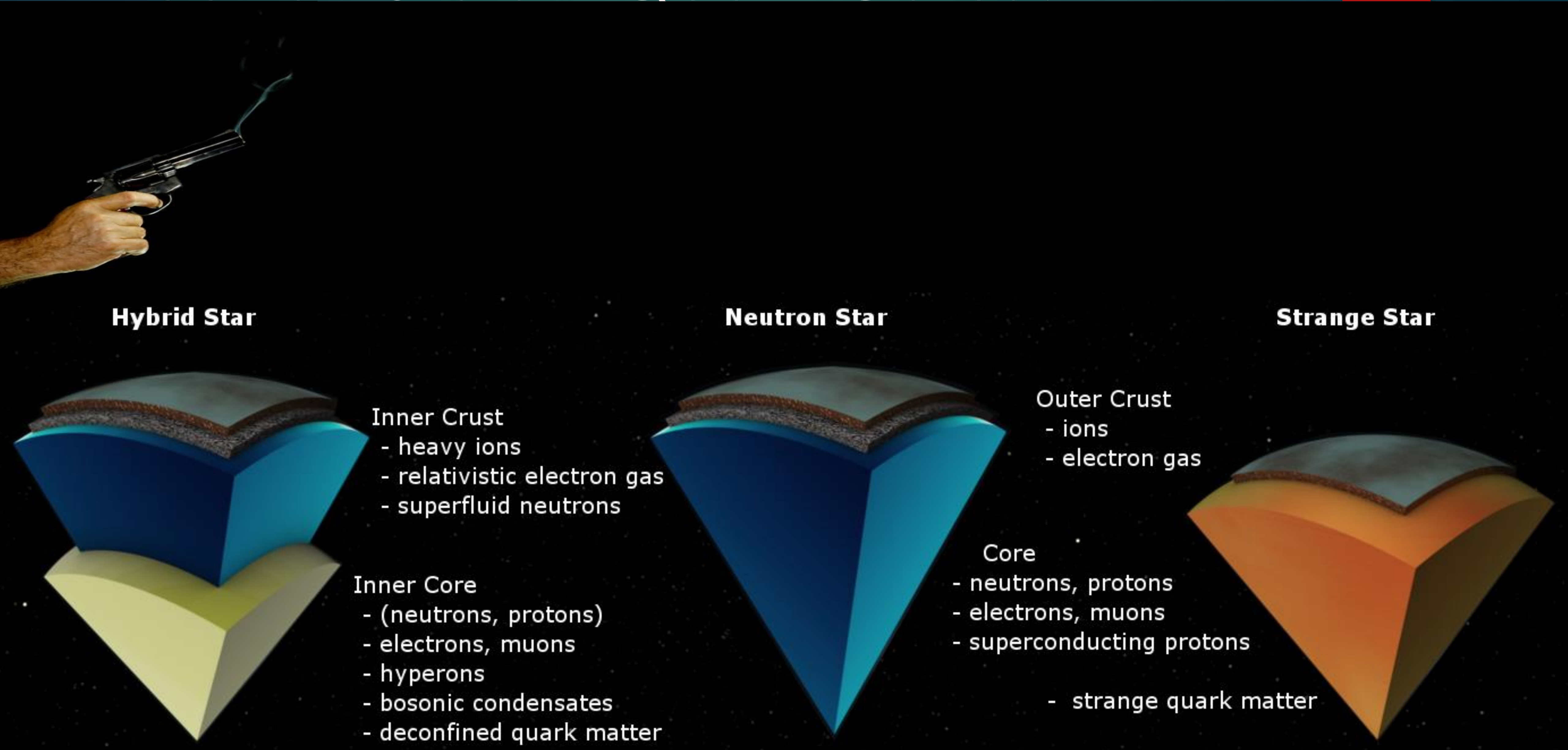

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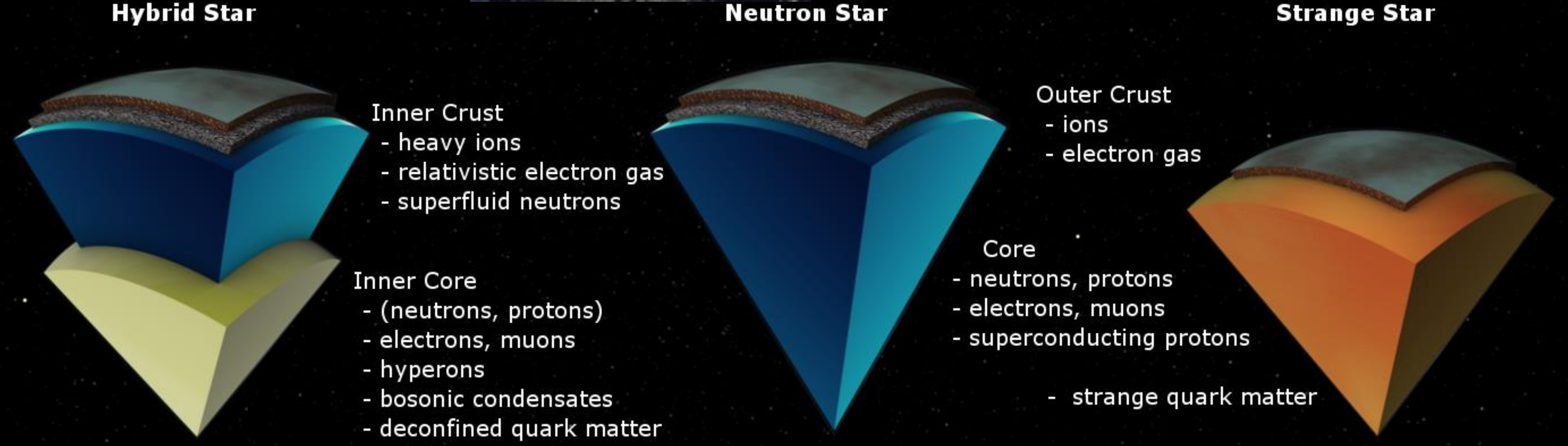

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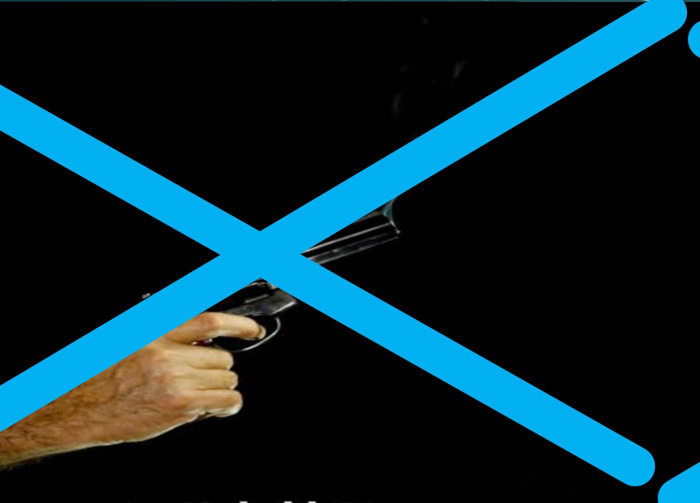
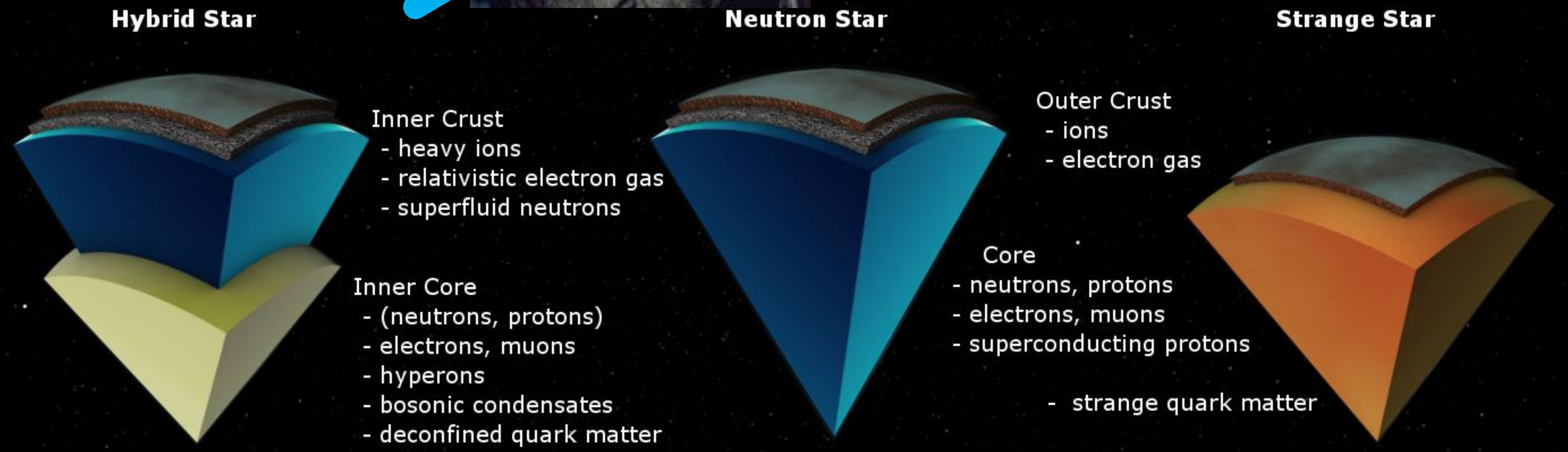




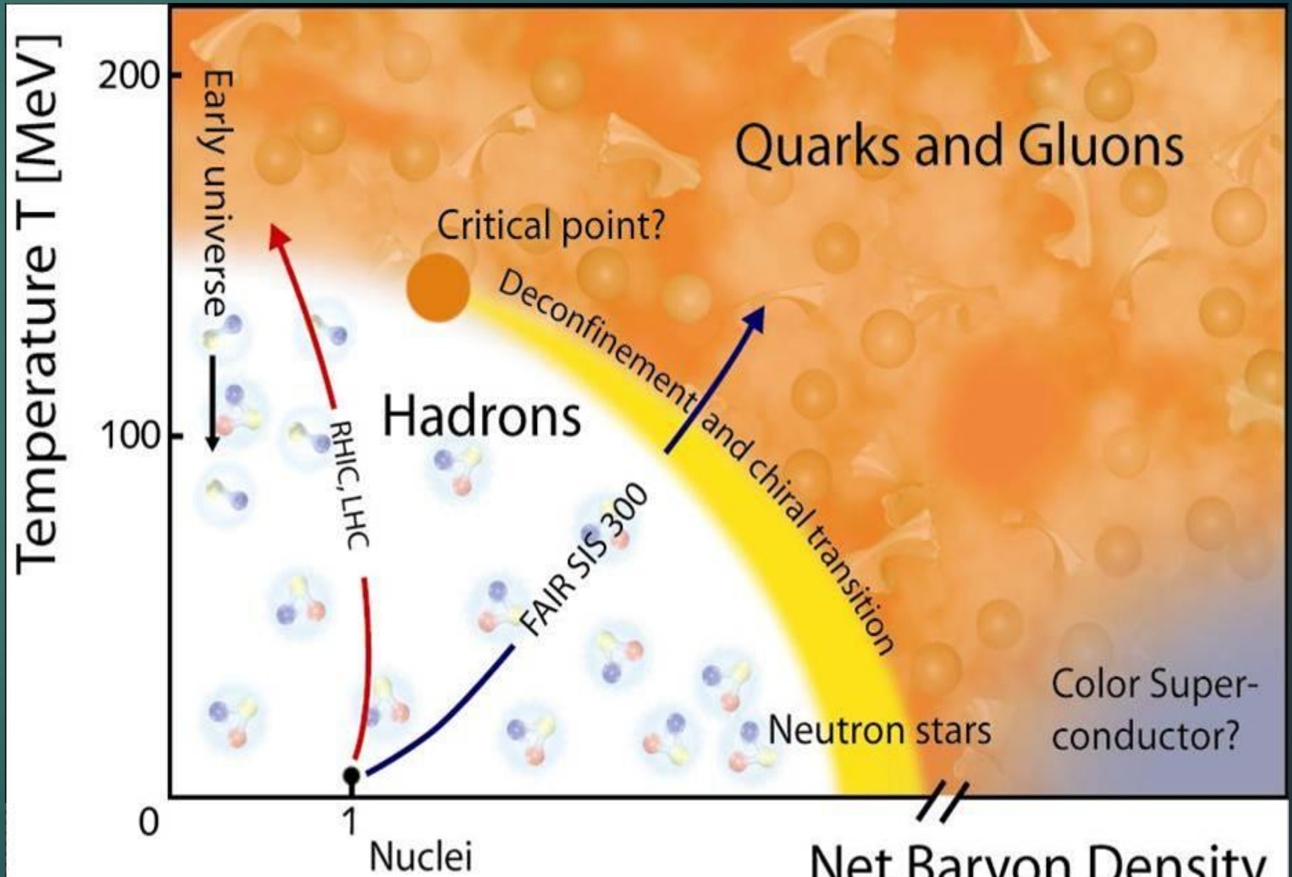
QCD Phase Diagram

- **dense hadronic matter**

<u>HIC in collider experiments</u>
Won’t cover the whole diagram
Hot and ‘rather’ symmetric

<u>NS as a 2nd accessible option</u>
Cold and ‘rather’ asymmetric

Problem is more complex than
It looks at first gaze



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QCD Phase Diagram

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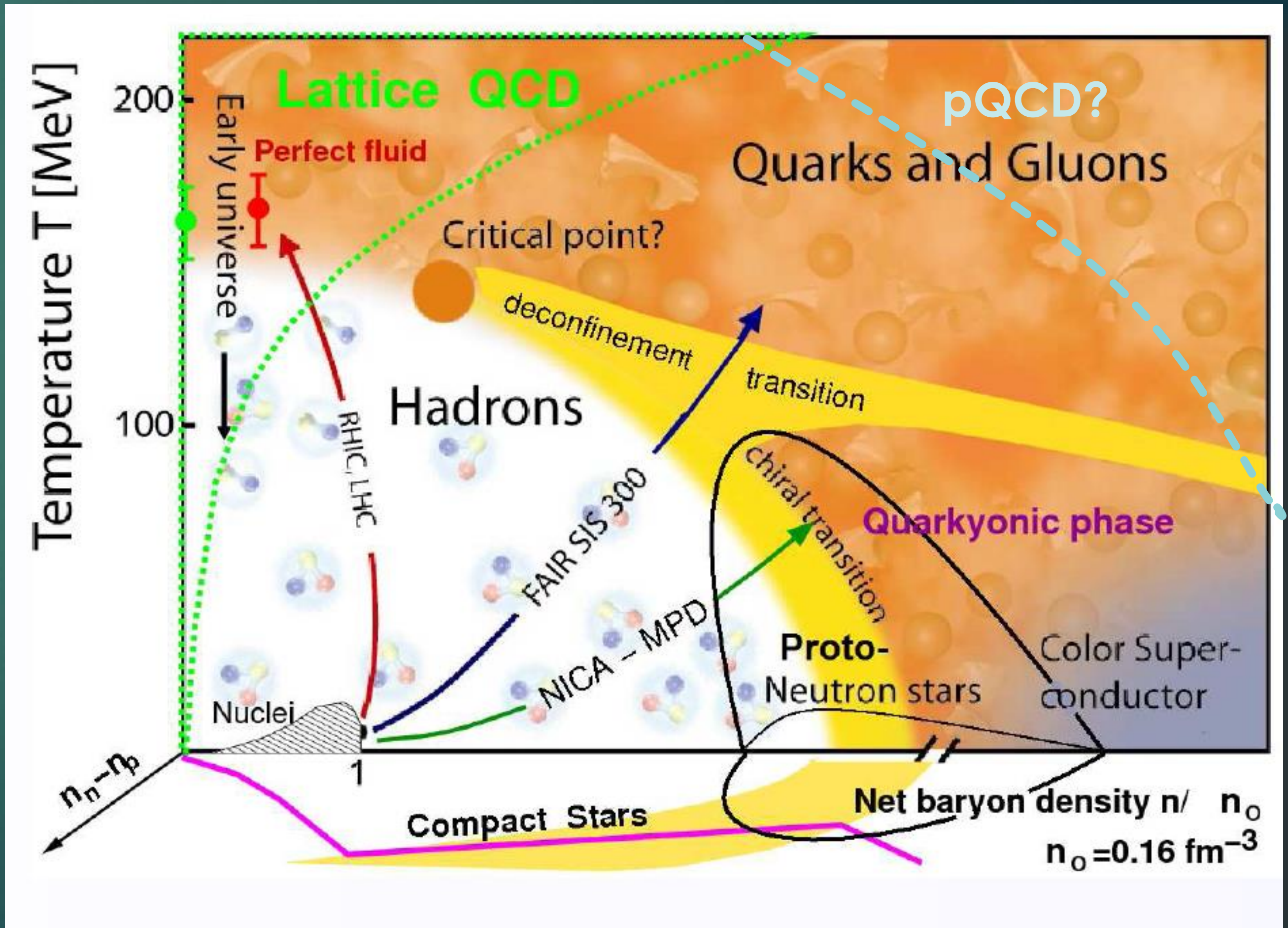
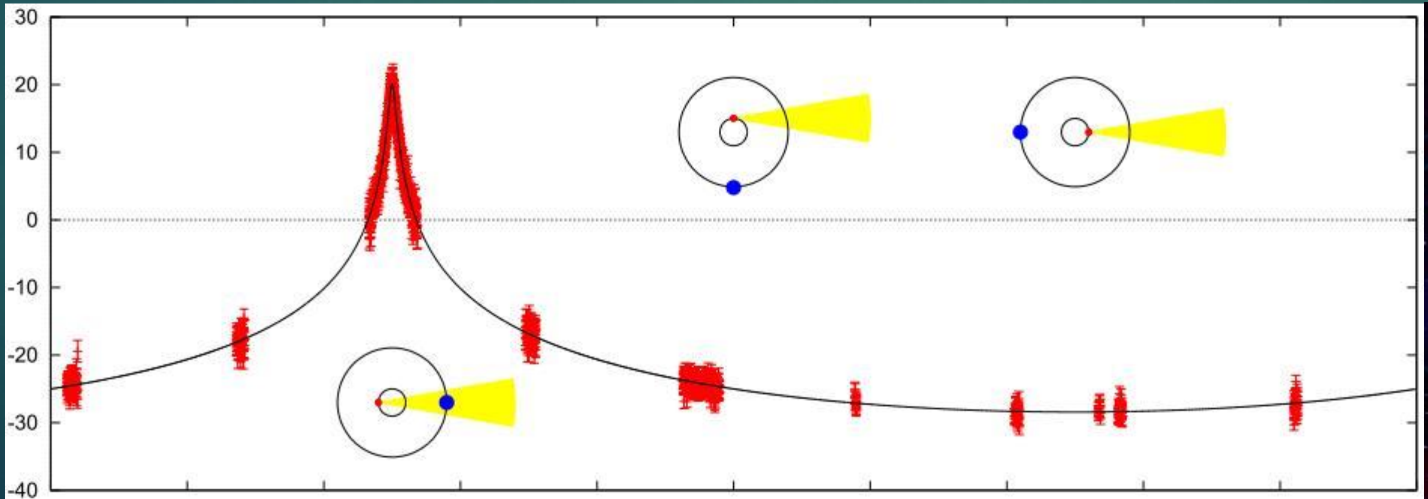
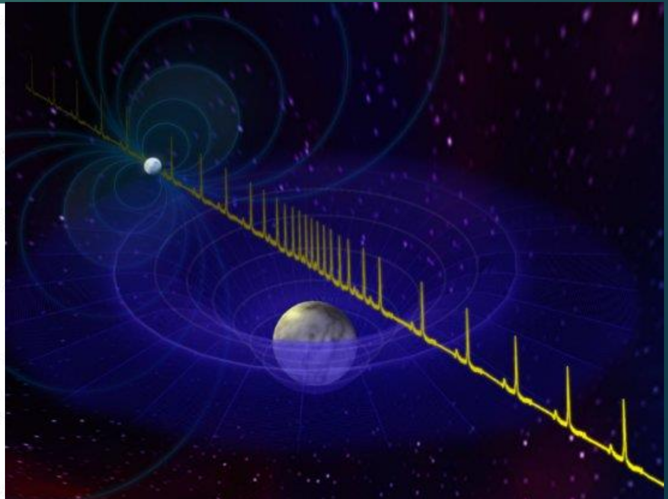


Neutron Star Data

- Data situation in general terms is good (masses, temperatures, ages, frequencies)
- Ability to explain the data with different models in general is good, too.

  ... sounds good, but becomes tiresome if everybody explains everything …
- For our purpose only a few observables are of real interest
- Most promising: High Massive NS with 2 solar masses (Demorest et al., Nature 467, 1081-1083 (2010))





Space, time and matter are related via Einsteins Field Equations

$$G_{\mu\nu} = -8\pi G T_{\mu\nu}$$

Einstein Tensor $G_{\mu\nu}$

defined by metric

Energy Momentum Tensor $T_{\mu\nu}$

defined by equation of state

Approximations

non rotating, spheric symmetry

$$ds^2 = g_{\mu\nu} dx^\mu dx^\mu$$

$$\longrightarrow g_{00}(r)dt^2 + g_{11}(r)dr^2 + g_{22}(r)d\theta^2 + g_{33}(r,\theta)d\phi^2$$

hydrostatic equilibrium

$$-pg^{\mu\nu} + (p+\varepsilon)u^\mu u^\nu$$

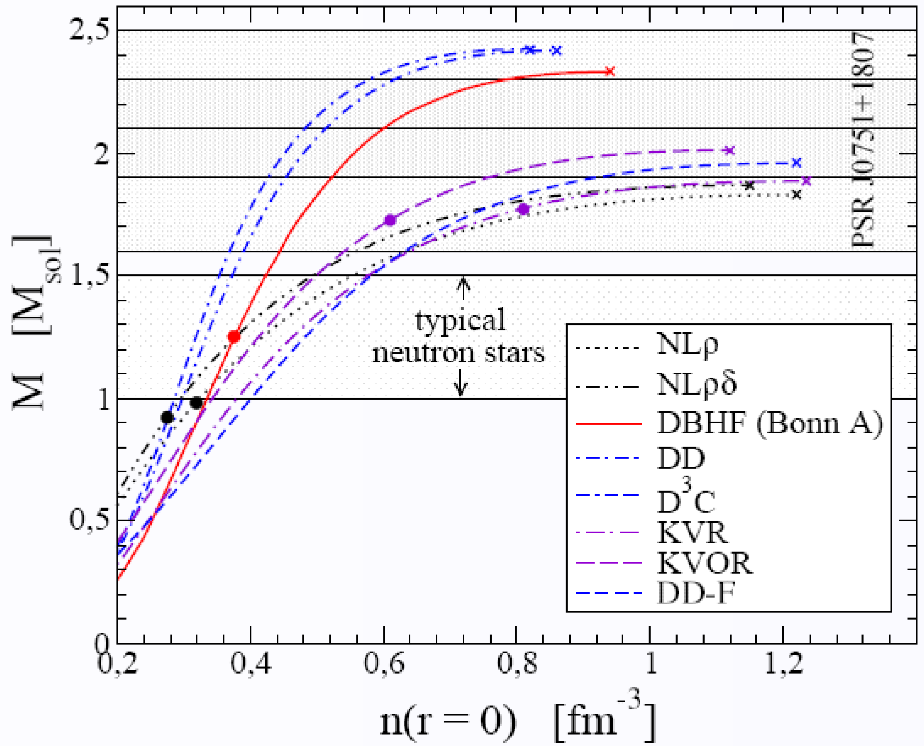
Tolman-Oppenheimer-Volkov (TOV) Equations (1939)

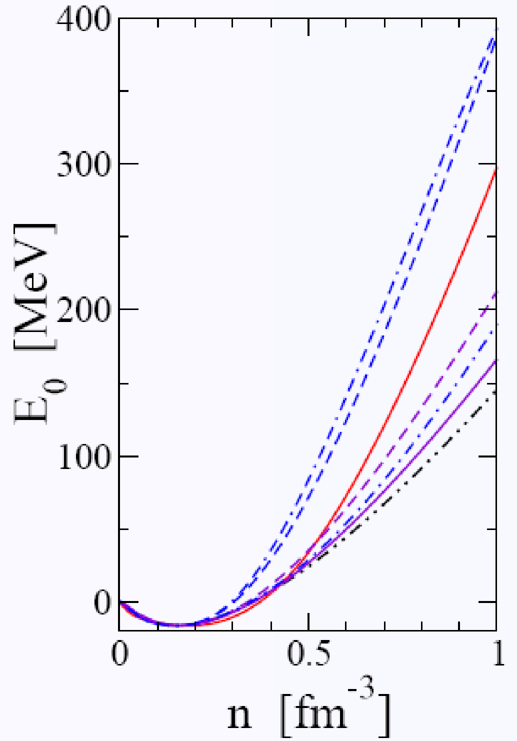
$$\frac{dp(r)}{dr} = -\frac{Gm(r)\varepsilon(r)}{r^2}\left(1+\frac{p(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi r^3 p(r)}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

$$m(r) = 4\pi\int_0^r dr'\, r'^2 \varepsilon(r')$$

NS masses and the (QM) Equation of State

- NS mass is sensitive mainly to the sym. EoS (In particular true for heavy NS)
- Folcloric:
  QM is soft, hence no NS with QM core
- Fact:
  QM is softer, but able to support QM core in NS
- Problem:
  (transition from NM to) QM is barely understood





$M(n)$ correlated to $E_0(n)$

stiff: higher M_{max} at smaller densities

soft: smaller M_{max} at higher densities

Quark Matter

What is so special about quarks?

Confinement: No isolated quark has ever been observed
<u>Quarks are confined</u> in baryons and mesons

Dynamical Mass Generation:

Proton 940 MeV, 3 constituent quarks with each 5 MeV
→ 98.4% from .... somewhere?

and then this:
eff. quark mass in proton: 940 MeV/3 ≈ 313 MeV
eff. quark mass in pion : 140 MeV/2 = 70 MeV

quark masses generated by interactions only
‚out of nothing'
interaction in QCD through (self interacting) gluons
<u>dynamical chiral symmetry breaking</u> (DCSB)
is a distinct <u>nonperturbative</u> feature!

Confinement and DCSB are connected. Not trivially seen from QCD Lagrangian.
Investigating quark-hadron phase transition requires nonperturbative approach.

Quark Matter

Confinement and DCSB are features of QCD.
It would be too nice to account for these phenomena
when describing QM in Compact Stars...

Currently used approaches to describe dense QM:

Bag-Model :

While Bag-models certainly account for confinement (constructed to do exactly this)
they do not exhibit DCSB (quark masses are fixed - bare quark masses).

Chodos, Jaffe et al: Baryon Structure (1974)
Farhi, Jaffe: Strange Matter (1984)

NJL-Model :

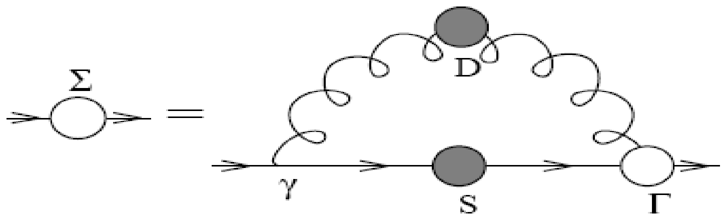
While NJL-type models certainly account for DCSB (applied, because they do)
they do not (trivialy) exhibit confinement.
Modifications to address confinement exist (e.g. PNJL) but are not entirelly satisfying
Both models: Inspired by, but not originally based on QCD.

Nambu, Jona-Lasinio (1961)

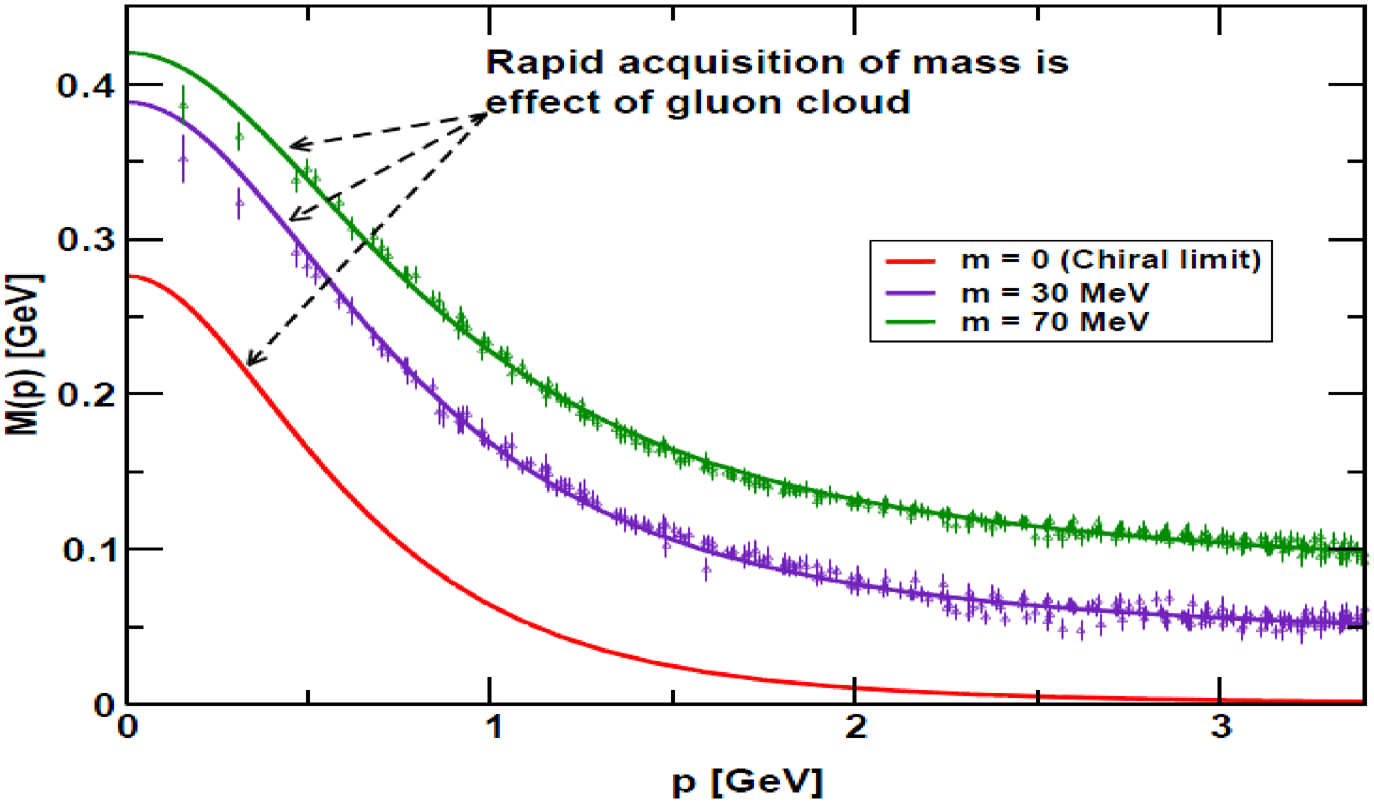
Lattice QCD still fails at T=0 and finite μ

Dyson-Schwinger Approach

Derive gap equations from QCD-Action. Self consistent self energies.
Successfuly applied to describe meson and hadron properties
Extension from vacuum to finite densities desirable
→ EoS within QCD framework



DSE : dynamical, momentum dependent mass generation



momentum dep. (here @ T=μ=0)
LQCD as benchmark

Neither NJL nor BAG have this!

How do momentum dependent
gap solutions affect
- EoS of deconfined quark matter?
- EoS of confined quark matter?
- transport properties in medium?

Roberts (2011)
Bhagwat et al. (2003,2006,2007)
P. O. Bowman et al. (2005)

Bag model: bare quark mass ~5 MeV at all densities
NJL model: constant quark mass at all momenta, but changing dynamically with density

Quark Matter

Confinement and DCSB are features of QCD.
It would be too nice to account for these phenomena when describing QM in Compact Stars...

Current reality is:

Bag-Model :

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Modifications to address these shortcomings exist (e.g. PNJL)
Still holds: Inspired by, but not based on QCD.

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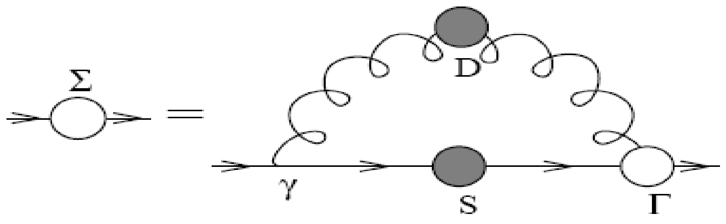


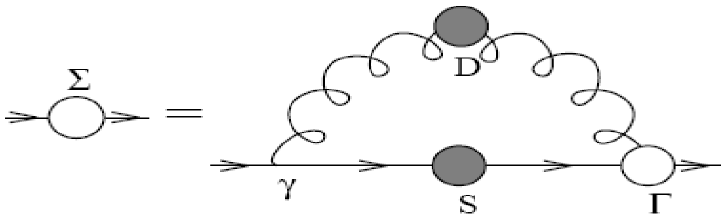
→ **THIS TALK: Bag and NJL model are simple limits within DS approach**

Dyson Schwinger Perspective

One particle gap equation(s)

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p} + i\gamma_4(p_4 + i\mu) + m + \Sigma(p;\mu)$$

Self energy -> entry point for simplifications



$$\Sigma(p;\mu) = \int_\Lambda \frac{d^4q}{(2\pi)^4} g^2 D_{\rho\sigma}(p-q)\gamma_\rho \frac{\lambda^a}{2} S(q)\Gamma^a_\sigma(p;q)$$

General (in-medium) gap solutions

$$S^{-1}(p;\mu) = i\vec{\gamma}\vec{p}A(p;\mu) + i\gamma_4(p_4 + i\mu)C(p;\mu) + B(p;\mu)$$

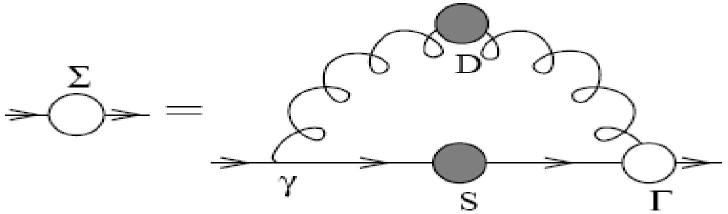
DSE -> NJL model

$$g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2}\delta_{\rho\sigma},$$

Gluon contact interaction in configuration space (other models exist)

$$\Gamma^a_\rho(p;q) = \frac{\lambda^a}{2}\gamma_\rho.$$

Rainbow approximation



$$\vec{p}^2 A_p = \vec{p}^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\vec{p}\vec{q}A_q}{\vec{q}^2A_q^2+\widetilde{q}_4^2C_q^2+B_q^2},$$

$$A = 1$$

$$B_p = m + \frac{16N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{B_q}{\vec{q}^2A_q^2+\widetilde{q}_4^2C_q^2+B_q^2},$$

$$B_\mu = m + \frac{4N_c}{9m_G^2}n_s(T,\mu^*,B),$$

$$\widetilde{p}_4^2 C_p = \widetilde{p}_4^2 + \frac{8N_c}{9m_G^2}\int_\Lambda \frac{d^4q}{(2\pi)^4}\frac{\widetilde{p}_4\widetilde{q}_4C_q}{\vec{q}^2A_q^2+\widetilde{q}_4^2C_q^2+B_q^2},$$

$$\mu = \mu^* - \frac{2N_c}{9m_G^2}n_v(T,\mu^*,B),$$

$$\widetilde{p}_4 C = p_4 + i(\mu+\omega_\mu) \equiv \hat{p}_4$$

Thermodynamical Potential

DS: steepest descent

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

$$P_I = -\frac{1}{2}\mathrm{Tr}\Sigma S = \frac{3}{4} m_G^2 \omega_\mu^2 - \frac{3}{8} m_G^2 \phi_\mu^2$$

Compare to NJL type model with following Lagrangian (interaction part only):

$$\mathcal{L}_{\mathrm{I}} = \mathcal{L}_{\mathrm{S}} + \mathcal{L}_{\mathrm{V}} = G_s \sum_{a=0}^{8} (\bar{q}\tau_a q)^2 + G_v (\bar{q} i \gamma_0 q)^2.$$

$$\Omega_q = \Omega_q^0 + \frac{\phi^2}{4G_s} - \frac{\omega^2}{2G_v} - \Omega_q(T = \mu = 0)$$

$$\phi_\mu = \quad 2G_s N_c n_s(T, m_f^*, \mu_f^*)$$
$$\omega_\mu = -2G_s N_c n_v(T, m_f^*, \mu_f^*)$$

$$\frac{\partial \Omega_q}{\partial \phi_\mu} = \frac{\partial \Omega_q}{\partial \omega_\mu} = 0.$$

Thermodynamical Potential

DS: steepest descent
$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S].$$

NJL model is easily understood as a particular approximation of QCD's DS gap equations

$$P_{FG} = \mathrm{Tr}\ln S^{-1} = 2N_c \int_\Lambda \frac{d^4p}{(2\pi)^4} \ln(\vec{p}^2 + \hat{p}_4^2 + B_\mu^2)$$

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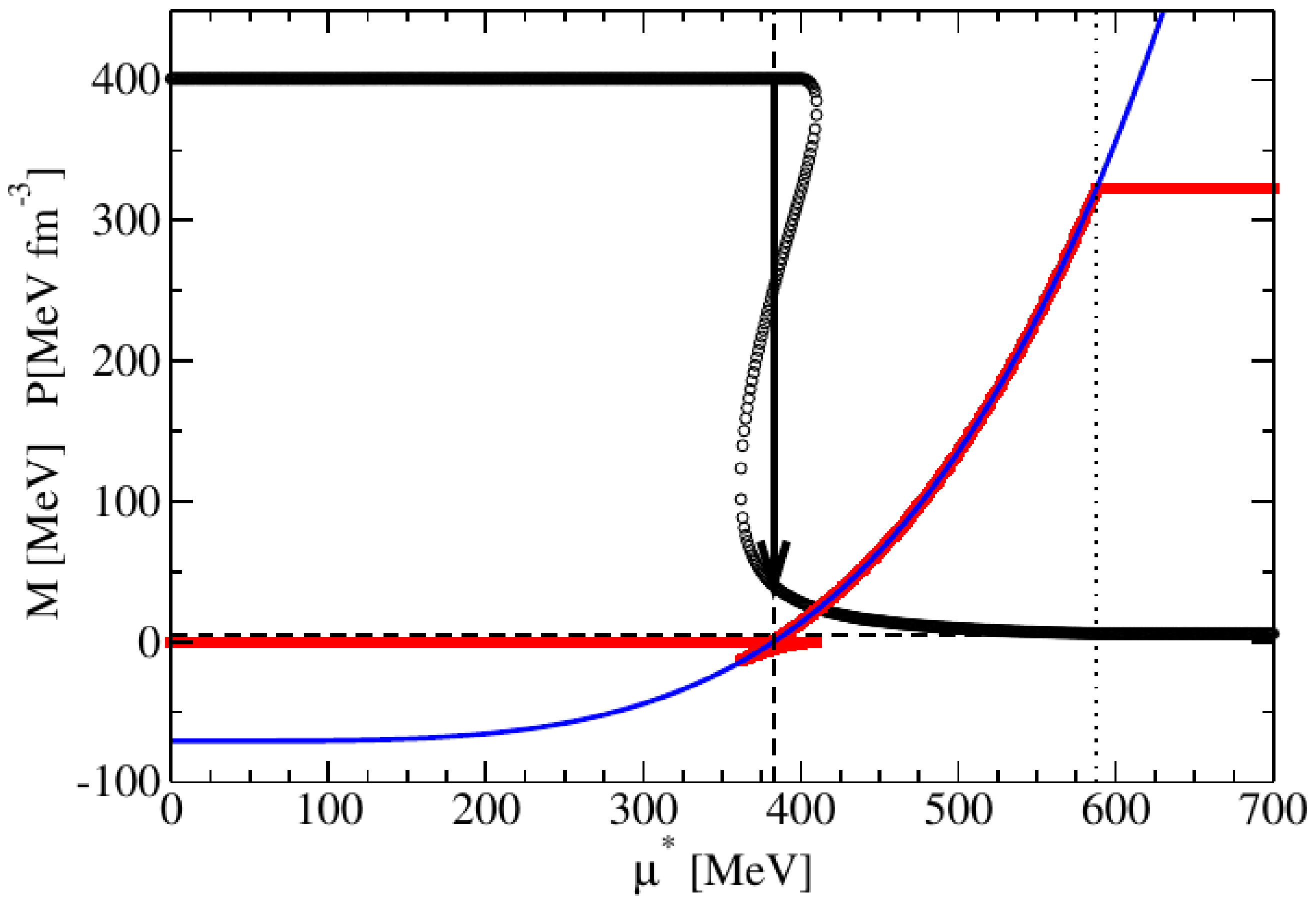
$$\frac{\partial\Omega_q}{\partial\phi_\mu} = \frac{\partial\Omega_q}{\partial\omega_\mu} = 0.$$

Bag Model from NJL perspective

obvious differences between NJL and Bag:

- **DχSB**
- confinement
- vector interaction



u,d-quark

Mass

Pressure NJL

Pressure Ideal Gas - Bag

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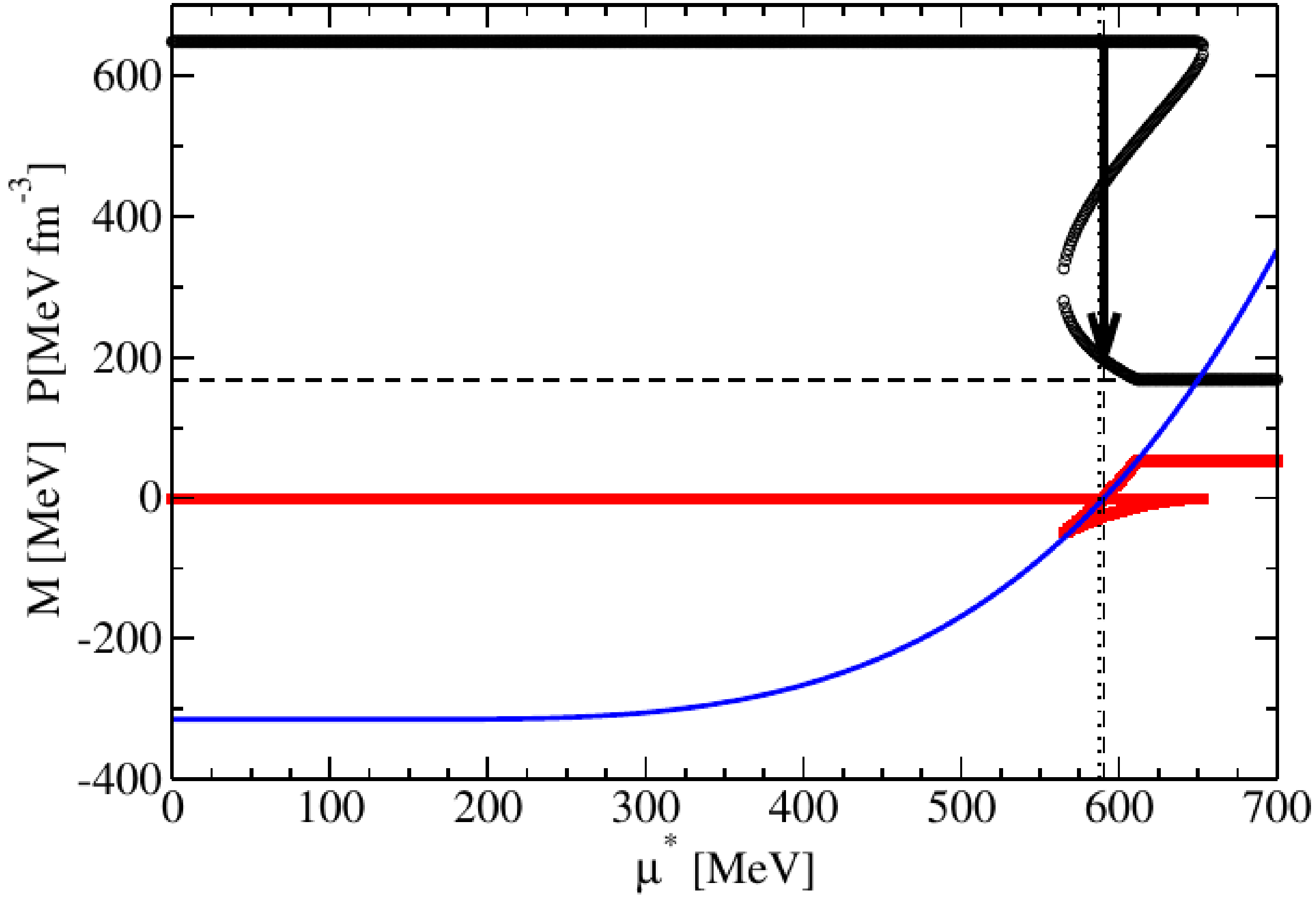

s-quark

Mass

Pressure NJL

Pressure Ideal Gas - Bag

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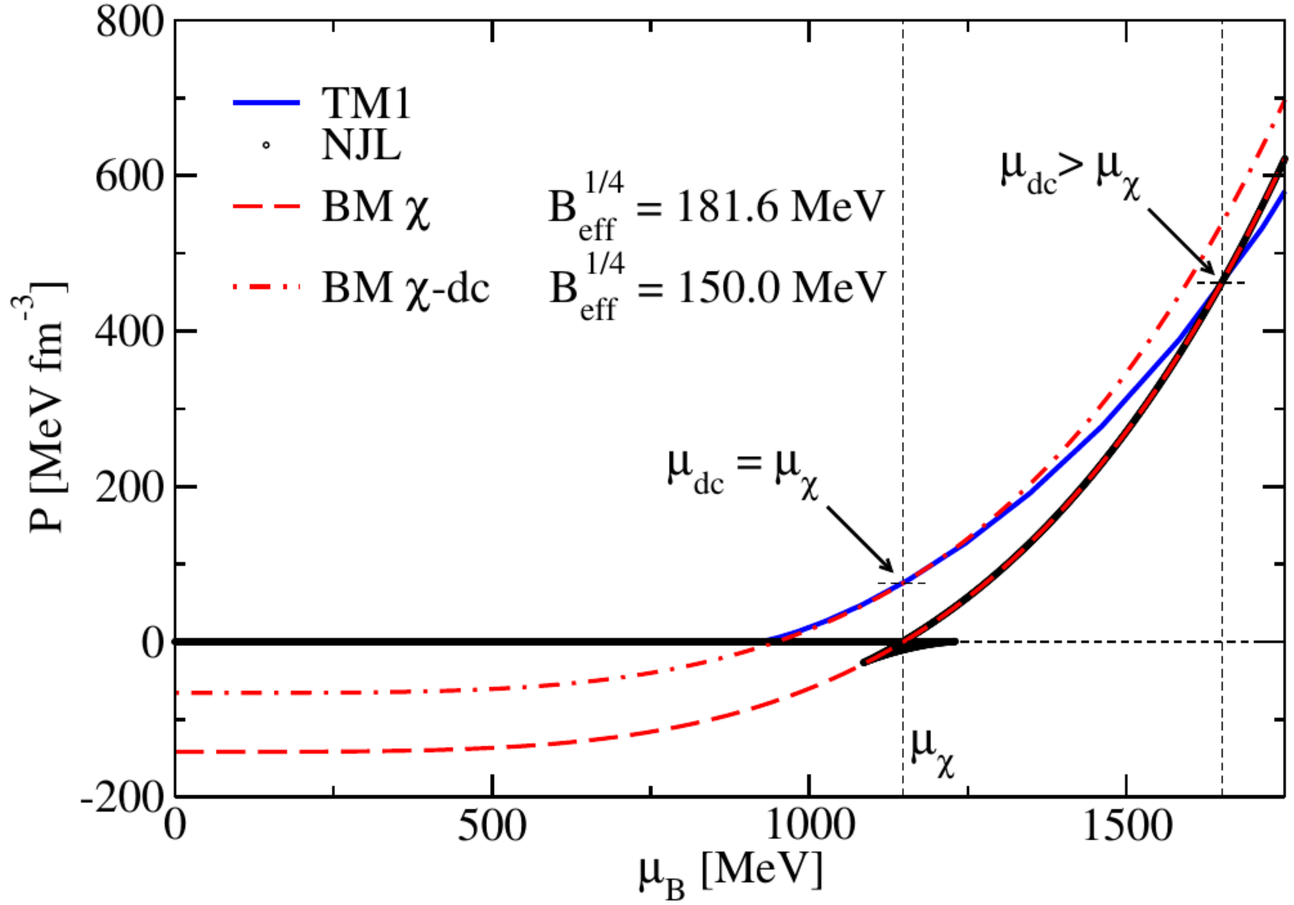
- DχSB
- **confinement**
- vector interaction



confinement

Pressure Quark NJL/Bag

Pressure Nuclear Matter

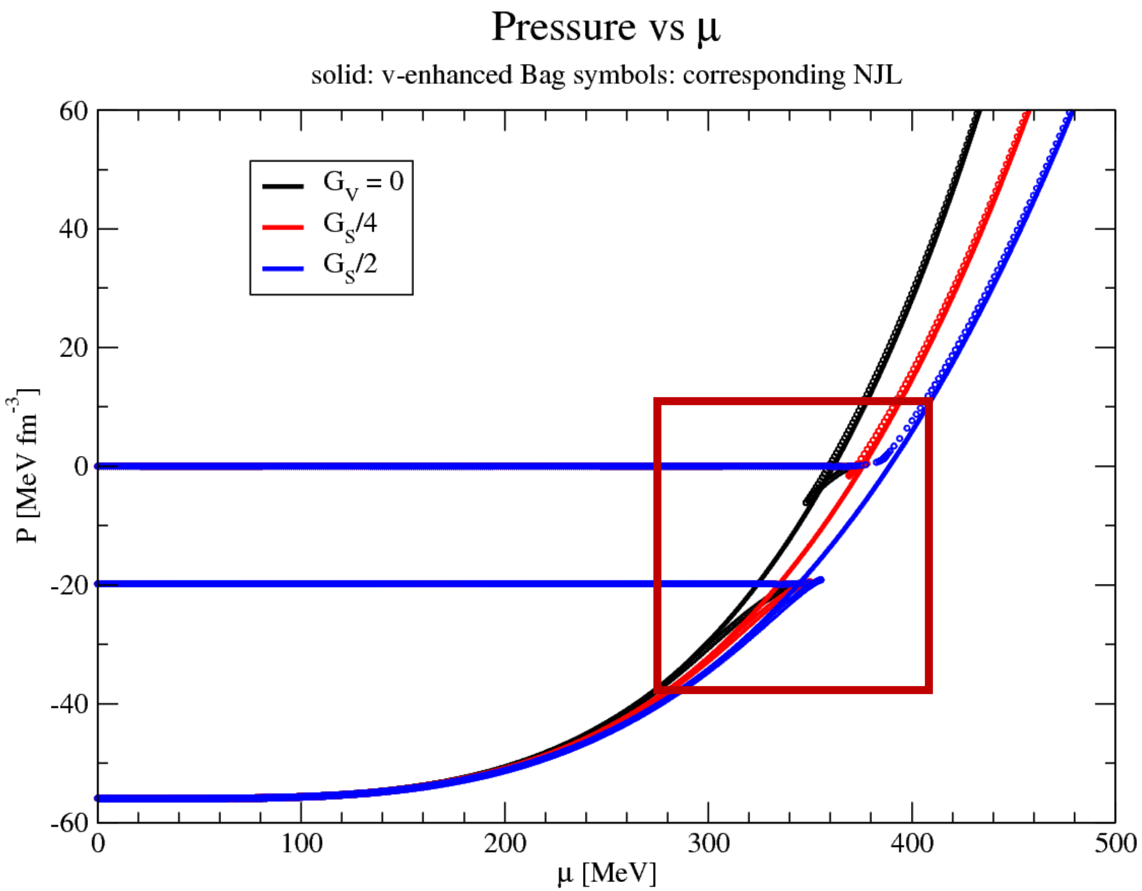
Obviously not zero at χ transition

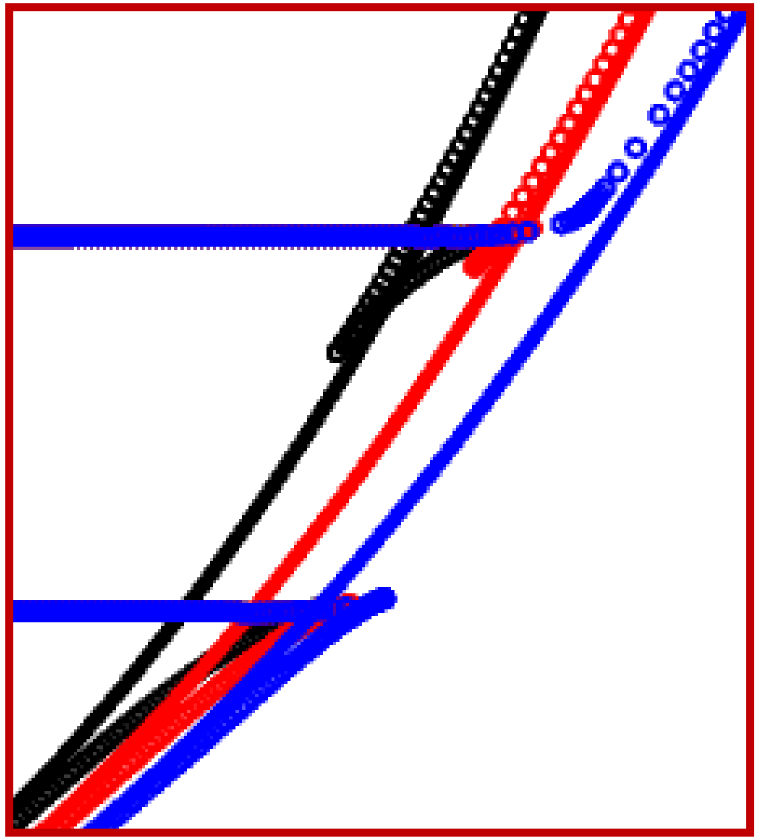
Reduce χ bag pressure – by hand

Bag Model from NJL perspective

obvious differences between NJL and Bag:

- DχSB
- confinement
- **vector interaction**





vBag: vector interaction enhanced bag model

Chiral + Vector:

$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

'Confinement':

$$P = \sum_f P_f^{kin} - B_{eff} \text{ with } B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

And, of course, chiral+vector+'confinement': Klahn & Fischer arXiv:1503.07442

Conclusions Part I

Vector enhanced bag like model can be derived from NJL - which can be obtained from DS gap equations

Bag model character: bare quark masses
chiral bag pressure as consequence of DχSB

Difference: chiral bag pressure
still no confining bag pressure
flavor dependent chiral bag constants
accounts for vector interaction -> promising for astrophysical applications

Beyond NJL: bag pressure due to deconfinement -> added by hand without harm to consistence

Advantage of the model: extremely simple to use, no regularization required

$$P^i_{BM}(\mu_i) = P_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) - P^i_{BAG}$$

$$\varepsilon^i_{BM}(\mu_i) = \varepsilon_{kin}(\mu_i^*) + \frac{K_v}{2} n_v^2(\mu_i^*) + P^i_{BAG}$$

$$\mu_i = \mu_i^* + K_v n_v(T, \mu_i^*)$$

Conclusions Part II

vBag: Bag-like model to reinvestigate … ‘everything’ … adding DχSB and vector interaction
application as simple as for the original bag model which omits these features

Neutron Stars

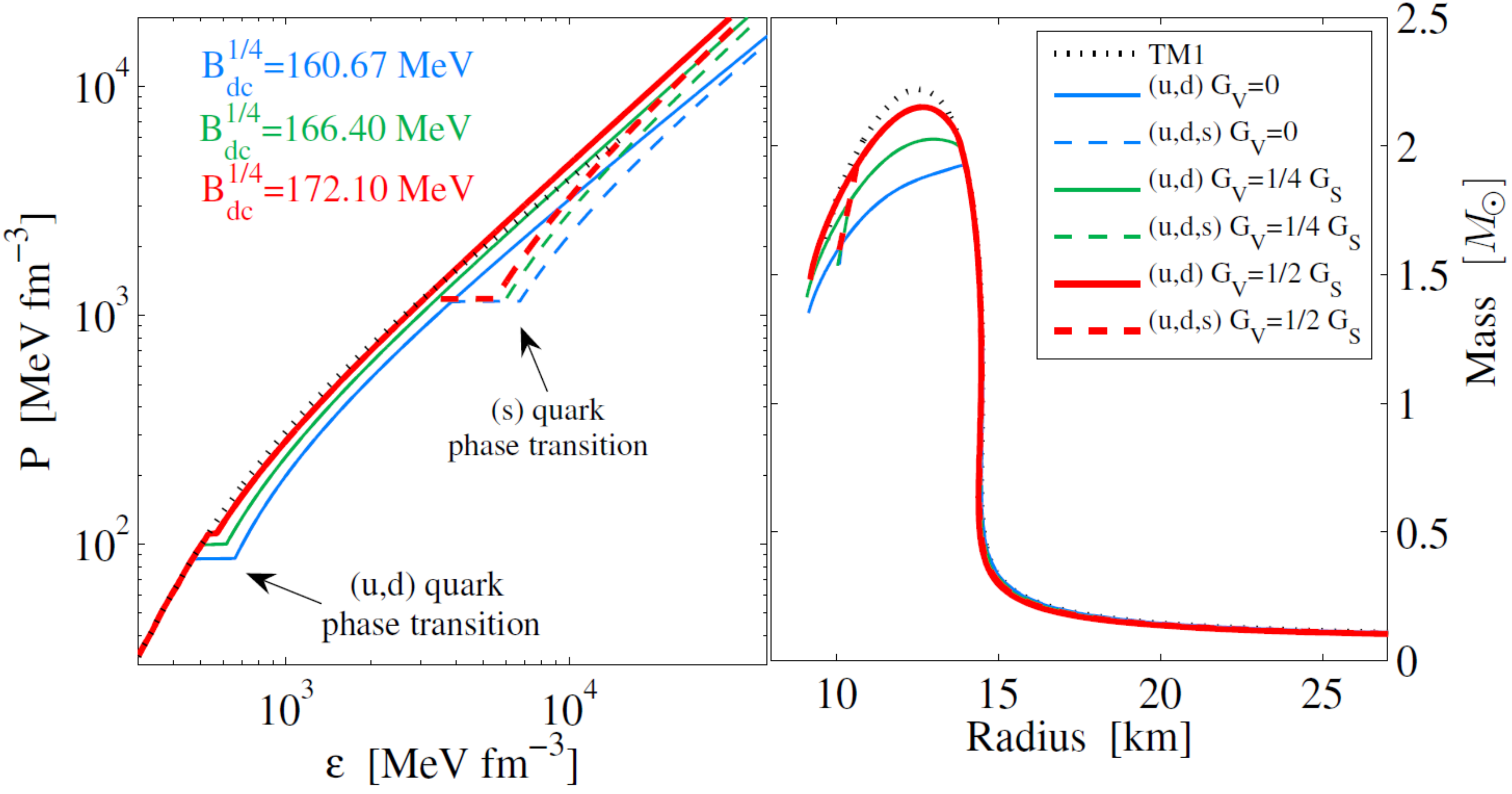
Mass Twin Solutions
Bayesian Analyses
Supernovae Simulations

Strange Matter

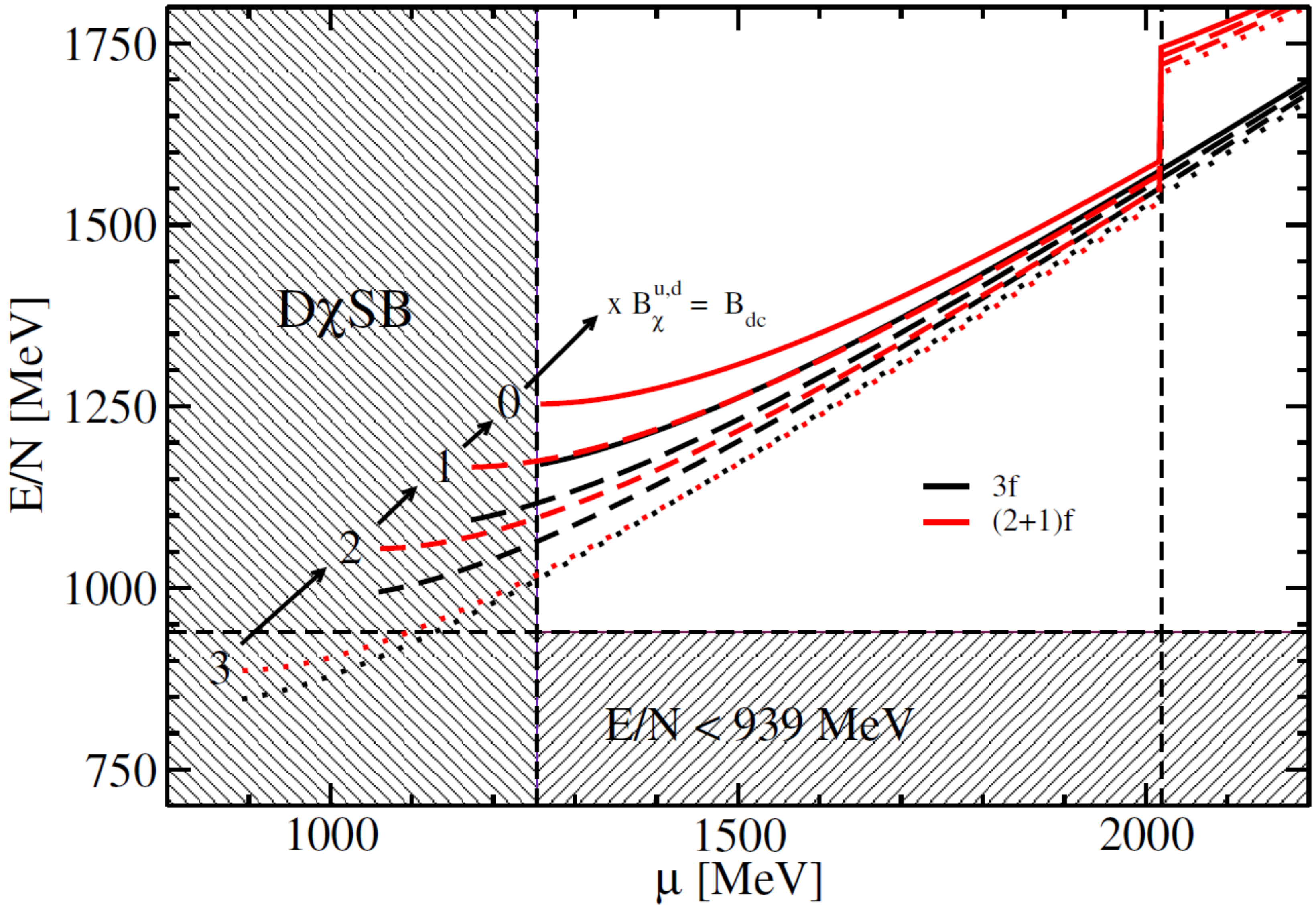
Studies of isospin dependence
Heavy Ion Collisions
Critical Point

(work in progress)

Neutron Stars with QM core – vBAG vs BAG



Absolutely Stable Strange Matter?



Original BAG models prediction of absolutely stable strange quark matter for certain bag constants is an artefact of neglecting dynamical chiral symmetry breaking ('BAG quarks' have bare quark mass)

Chodos et al have been aware of this simplification!

NJL model and DS studies do not confirm ASSM hypothesis.

vBAG accounts for DχSB

Conclusions Part III

vBAG:

- vector interaction resolves the problem of too soft bag model EoS w/o perturbative corrections
- No problem at all to obtain stable hybrid neutron star configurations
- Standard BAG models bag constant is understood to mimic confinement, DχSB is absent
- vBAG introduces effective bag constant with similar values to original BAG

$$B_{eff} = \sum_f B_\chi^f - B_{\rm dc}$$

- However, positive value due to chiral transition, deconfinement actually reduces B
- Absolutely stable strange matter likely ruled out due to DχSB

- NJL and Bag model result from particular approximations within Dyson-Schwinger approach

  rainbow approximation (quark-gluon vertex) + contact interaction (gluon propagator)
- Consequence: both models lack momentum dependent gap solutions

Effective gluon propagator

$$S(p;\mu)^{-1} = Z_2(i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4 + i\mu) + m_{\mathrm{bm}}) + \Sigma(p;\mu)$$

$$\Sigma(p;\mu) = Z_1 \int_q^\Lambda g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma^a_\sigma(q,p;\mu)$$

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

$$Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma^a_\nu(q,p) \to \int_q^\Lambda \mathcal{G}((p-q)^2)\, D^{\mathrm{free}}_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu$$

Specify behaviour o $\mathcal{G}(k^2)$

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D \delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda^2_{\mathrm{QCD}}\right)^2\right]} \mathcal{F}(k^2)$$

Infrared strength (zero width + finite width contribution)

running coupling for large k

EoS (finite densities):

1st term (Munczek/Nemirowsky (1983)) delta function in momentum space → Klähn et al. (2010)

2nd term → Chen et al.(2008,2011)

NJL model: $g^2 D_{\rho\sigma}(p-q) = \frac{1}{m_G^2} \delta_{\rho\sigma}$ delta function in configuration space = const. In mom. space

Munczek/Nemirowsky -> NJL's complement

Wigner Phase

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]} \mathcal{F}(k^2)$$

$B_W = 0, A_W = C_W$:

$$C_W(p,\mu) = \frac{1}{2}\left(1 + \sqrt{1 + \frac{2\eta^2}{p_3^2 + (p_4 + i\mu)^2)}}\right)$$

Nambu Phase

$A_N = C_N$.

$\Re(\tilde{p}^2) < \frac{\eta^2}{4}$:

$$\begin{aligned} B_N(p,\mu) &= \sqrt{\eta^2 - 4(p_3^2 + (p_4 + i\mu)^2))} \\ C_N(p,\mu) &= 2 \end{aligned}$$

$\Re(\tilde{p}^2) > \frac{\eta^2}{4}$:

$$A_N = A_W, B_N = B_W, C_N = C_W.$$

<u>MN antithetic to NJL</u>

NJL:contact interaction in x

MN:contact interaction in p

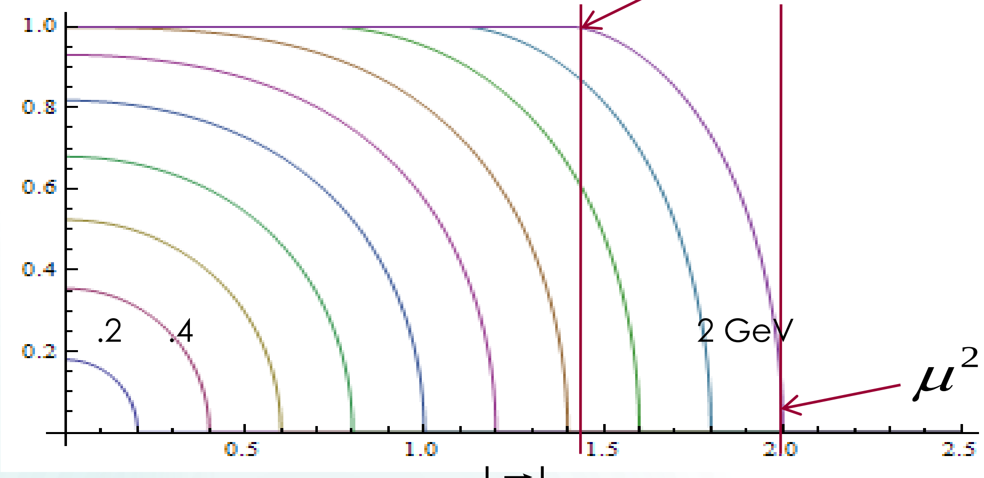
Munczek/Nemirowsky

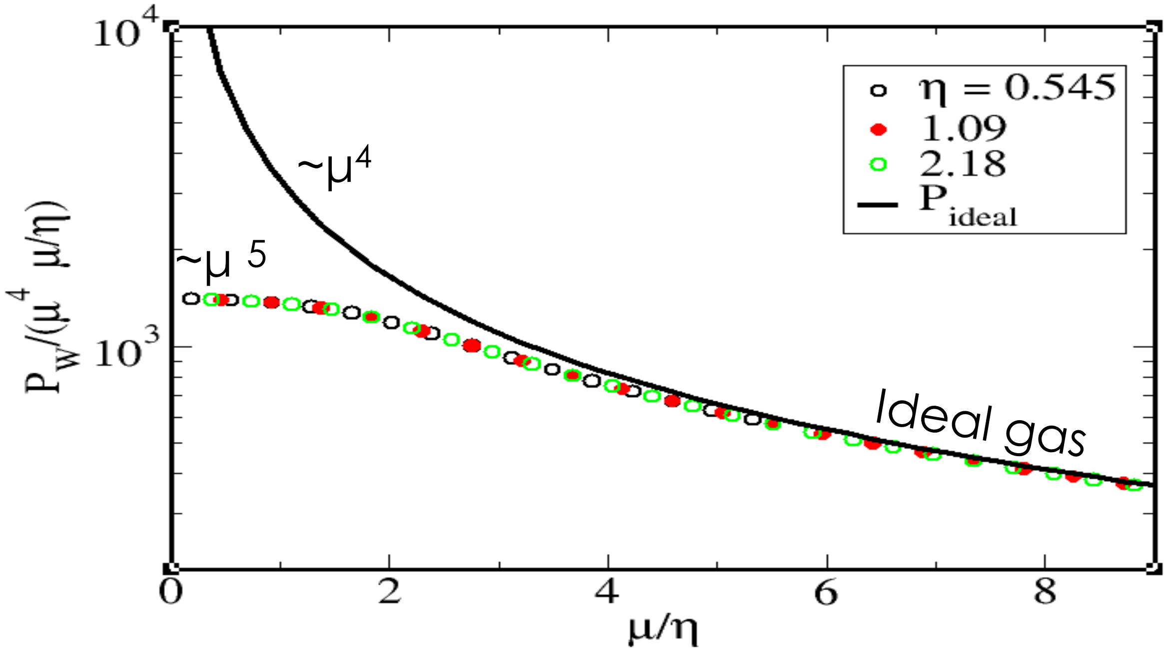
$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr_D}(-\gamma_4)S(p;\mu)$$

$$\boxed{P(\mu<\eta) = P_0 + \int_0^{\mu} d\mu'\, n(\mu') \propto P_0 + const\times\mu^5}$$

Wigner Phase

$\vec{p}^{\,2} = \mu^2 - 2\eta^2$





$\mu^2 \geq 2\eta^2$ to obtain $f_1(\vec{p}^{\,2}=0)=1$ model is scale invariant regarding μ/η

$P(\mu)\propto\mu^5$ well satisfied up to $\mu/\eta\approx 1$

$(\eta = 1.09\,\mathrm{GeV})$

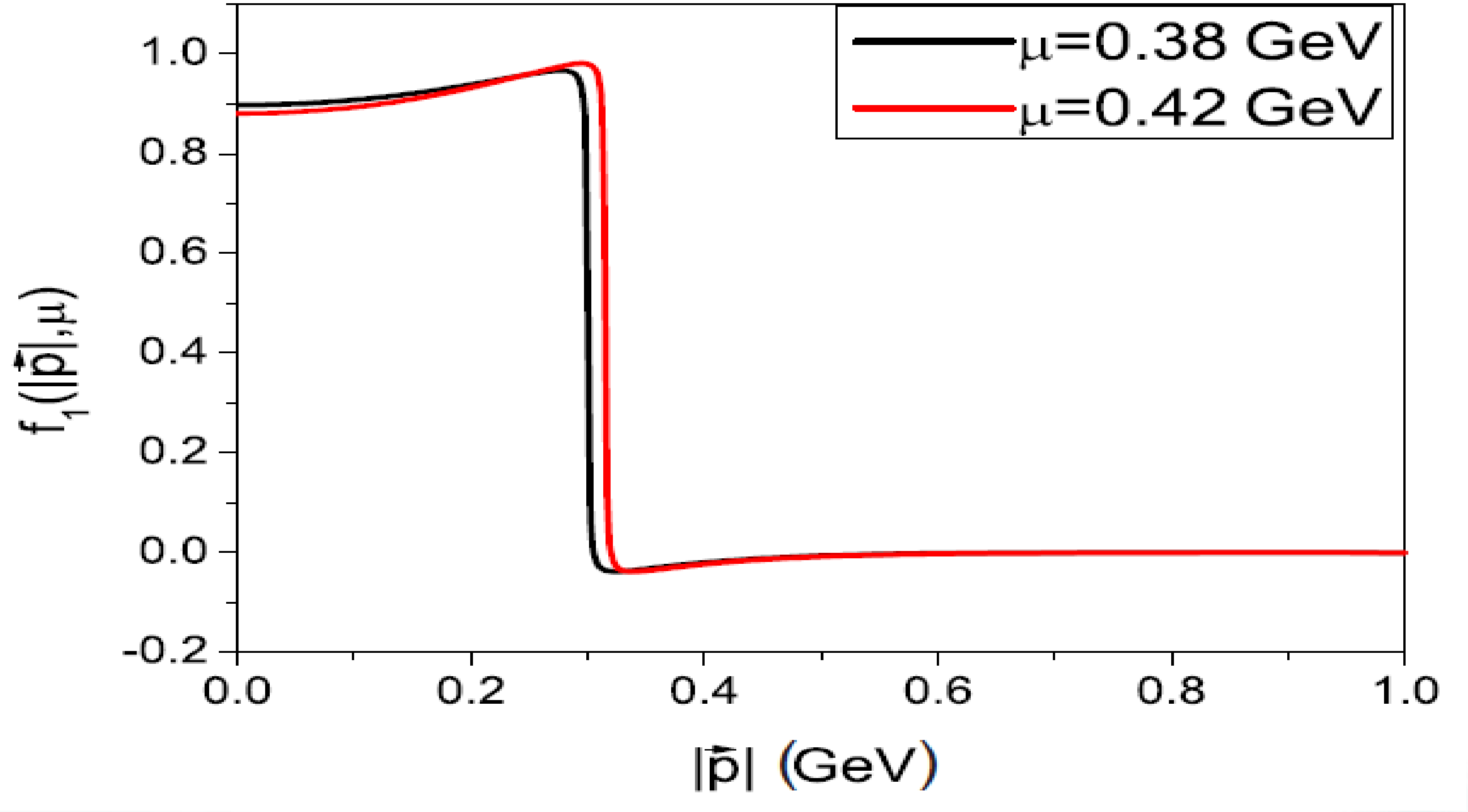
‚small' chem. Potential: $f_1(\vec{p}^{\,2}=0,\mu<\eta)\propto\mu \leftarrow$ $\boxed{n(\mu<\eta) = \frac{2N_cN_f}{2\pi^2}\int d^3\vec{p}\ f_1(|\vec{p}|)\propto\mu^4}$

T. Klahn, C.D. Roberts, L. Chang, H. Chen, Y.-X. Liu PRC 82, 035801 (2010)

DSE – simple effective gluon coupling

$$\frac{\mathcal{G}(k^2)}{k^2} = 8\pi^4 D\delta^4(k) + \frac{4\pi^2}{\omega^6} D k^2 \mathrm{e}^{-k^2/\omega^2} + 4\pi \frac{\gamma_m \pi}{\frac{1}{2}\ln\left[\tau + \left(1 + k^2/\Lambda^2_{\mathrm{QCD}}\right)^2\right]} \mathcal{F}(k^2)$$

Wigner Phase   Less extreme, but again, 1particle number density distribution different from free Fermi gas distribution



Chen et al. (TK) PRD 78 (2008)

Thank you!

Conclusions

QCD in medium (near critical line):

- Task is difficult
- Not addressable by LQCD
- Not addressable by pQCD
- DSE are promising tool to tackle non-perturbative in-medium QCD
- Qualitatively very different results depending on effective gluon coupling
- Bag model a simple limiting case of NJL model
- NJL model a simple contact interaction model in the gluon sector
- vBag connects them, other models exist