Particle production in time-dependent background

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Motivation

In cosmology particle production is a part of crucial processes determinig the history of the universe:

- leptogenesis
- baryogenesis
- preheating, reheating (see J.Martin's talk)
- non-thermal dark matter production

Simplest model: one scalar field evolving in time-dependent background

$$\begin{array}{c} \text{vacuum}_{\text{in}} = |0_{\text{in}}\rangle \\ \text{operators} \left(a_k^{\text{in}}, a_k^{\text{in}\,\dagger} \right) \\ \text{modes} \left(v_k^{\text{in}} \right) \end{array} \neq \begin{array}{c} \text{vacuum}_{\text{out}} = |0_{\text{out}}\rangle \\ \text{operators} \left(a_k^{\text{out}}, a_k^{\text{out}\,\dagger} \right) \\ \text{modes} \left(v_k^{\text{out}} \right) \end{array}$$

Bogoliubov transformation

These two sets of operators act in the same Hilbert space so we can express one using another

$$a_k^{\text{out}} = \alpha_k a_k^{\text{in}} + \beta_k a_k^{\text{in}\,\dagger}$$
$$a_k^{\text{out}\,\dagger} = \alpha_k^* a_k^{\text{in}\,\dagger} + \beta_k^* a_k^{\text{in}}$$

and calculate commutation relation in the new basis

$$[a_k^{\text{out}}, a_k^{\text{out}\,\dagger}] = [\alpha_k a_k^{\text{in}} + \beta_k a_k^{\text{in}\,\dagger}, \alpha_k^* a_k^{\text{in}\,\dagger} + \beta_k^* a_k^{\text{in}}] = ... = \Big(|\alpha_k|^2 - |\beta_k|^2\Big)[a_k^{\text{in}}, a_k^{\text{in}\,\dagger}].$$

Commutation relation is fixed so we obtain the normalization condition for Bogoliubov coefficients in case of the scalar field

$$|\alpha_k|^2 - |\beta_k|^2 = 1.$$

For fermions: $|\alpha_k|^2 + |\beta_k|^2 = 1$ because of the different form of commutation relation.

It turns out that the occupation number of produced particles can be represented as

$$n_k \equiv \langle 0^{\text{in}}|N_k|0^{\text{in}}\rangle = \langle 0^{\text{in}}|a_{\vec{k}}^{\text{out}\,\dagger} a_{\vec{k}}^{\text{out}}|0^{\text{in}}\rangle = V|\beta_k|^2.$$

It seems that if $\beta_k = 0$ particles are not produced.

Adiabaticity

What is the condition under which particle production occurs?

We choose adiabatic vacuum as it gives us the minimal production:

$$v_k \sim \frac{1}{\sqrt{\omega_k}} e^{\pm i \int \omega(t')dt'}.$$

There are two regimes to solve the equation for mode functions

$$\ddot{v}_{\vec{k}} + \omega^2_{\vec{k}}(t) v_{\vec{k}} = 0$$

- adiabatic region: $\dot{\omega}_k/\omega_k^2 < 1$

$$n_k(t) \approx \frac{\rho_k}{\omega_k} \approx \frac{|\dot{v}_k|^2}{\omega_k} \approx \frac{1}{\omega_k} |\sqrt{\omega e^{\pm i \int \omega}}|^2 \approx \text{const}$$

- non-adiabatic region: $\dot{\omega}_k/\omega_k^2 > 1$

$n_k(t) \neq \text{const}$ particle production occurs

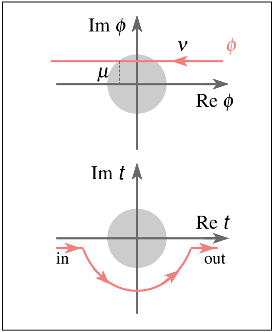
Simple model (L.Kofman et al., arXiv:hep-th/0403001)

$$V = \frac{1}{2} g^2 |\phi|^2 \chi^2$$

- asymptotically: $\langle \phi \rangle = vt + i\mu$, $\langle \chi \rangle = 0$
- non-adiabatic region: $|\phi| \lesssim \sqrt{v/g}$
- background field in non-adiabatic region: χ particles are produced
- produced particles induce a new linear potential

$$\rho_\chi = \int \frac{d^3k}{(2\pi)^3} n_k \sqrt{k^2 + g^2 |\phi(t)|^2} \approx g|\phi(t)| n_\chi$$

and an attractive force (''oscillations'')



- each time the occupation number of produced χ particles is:

$$n_k^\chi = V \cdot |e^{-i \int^t dt' \omega_k(t')}|^2 = V \cdot \exp\left(-\pi \frac{k^2 + g^2 \mu^2}{gv} \right)$$

Simple supersymmetric model

Superpotential:

$$W = \frac{g}{2}\Phi X^2$$

Potential:

$$V_{\text{scalar}} = \frac{g^2}{4}|\chi|^4 + g^2|\phi|^2|\chi|^2$$

Why supersymmetry?

- natural way of introducing fermions
- cancellation of UV divergences
- it's simple but still nontrivial (2 scalars + 2 fermions, 2 massive + 2 massless)

Once again we can choose: $\langle\phi\rangle = vt + i\mu$.

After one transition:

- $n_\phi \approx 0$, $n_{\psi_\phi} \approx 0$
- $n_{\psi_\chi} = 2\frac{(gv)^{3/2}}{(2\pi)^3}e^{-\pi g\mu^2/v}$
- $n_\chi^{\text{broken before}} = 2\frac{(gv)^{3/2}}{(2\pi)^3}e^{-\frac{\pi(g^2\mu^2+m^2)}{gv}}$
- $n_\chi^{\text{broken after}} = 2\frac{(gv)^{3/2}}{(2\pi)^3}e^{-\pi g\mu^2/v}$

Cut-off momentum (for larger k we leave the non-adiabatic region):

$$k_{\max}^2 = \frac{gv}{\pi}\ln(2) - g^2\mu^2$$

Simple model with small correction

Superpotential:

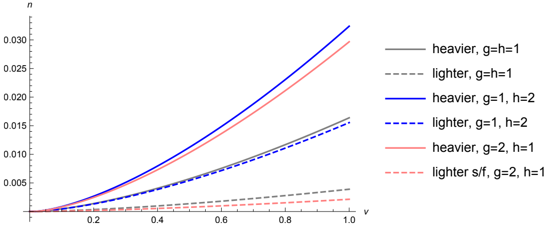
$$W = \frac{g}{2}\Phi X^2 + h\Phi\Psi X$$

Potential:

$$V_{\text{scalar}} = |g\chi + h\psi|^2|\phi|^2 + |\frac{g}{2}\chi + h\psi||\chi|^2 + h^2|\phi|^2|\chi|^2$$

Once again we can choose: $\langle\chi\rangle = \langle\psi\rangle = 0$ and $\langle\phi\rangle = vt + i\mu$.

Mass eigenstates are mixed (still: fermion's mass = scalar's mass).



Conclusion: heavier states are produced more efficiently.

Role of interactions

So far:
produced particles just propagate and cause backreaction (induced potential) but they do not interact.

What is the role of these interactions?
(in literature called rescattering)

Yang-Feldman equation

Let's take a scalar field Ψ with canonical commutation relation

$$[\Psi(t,\vec{x}), \dot{\Psi}(t,\vec{y})] = i\delta^3(\vec{x}-\vec{y})$$

and equation of motion of the form

$$\left(\partial^2 + M^2(x)\right)\Psi(x) + J(x) = 0.$$

Its solution is called Yang-Feldman equation

$$\Psi(x) = \sqrt{Z}\Psi^{as}(x) - iZ\int_{t_{as}}^{x^0} dy^0 \int d^3y[\Psi^{as}(x), \Psi^{as}(y)]J(y),$$

where the integral part plays the role of retarded potential and

$$\Psi(t^{as},\vec{x}) = \sqrt{Z}\Psi^{as}(t^{as},\vec{x}).$$

Generalized Bogoliubov transformation

Expanding our asymptotic field into mode functions allows us to get

$$a^{\text{out}}_{\vec{k}} = \alpha_k a^{\text{in}}_{\vec{k}} + \beta_k a^{\text{in}\,\dagger}_{-\vec{k}} - i\sqrt{Z}\int d^4x e^{-i\vec{k}\cdot\vec{x}}\Big(-\beta_k \Psi^{\text{in}}_k(x^0) + \alpha_k \Psi^{\text{in}\,*}_k(x^0)\Big) J(x),$$

what establishes the generalized Bogoliubov transformation with coefficients defined as some combination of Z, Ψ^{in}_k and Ψ^{out}_k with usual normalization.

Occupation number

Occupation number is now

$$n_k = \langle 0^{\text{in}} | a_{\vec{k}}^{\text{out}\,\dagger} a_{\vec{k}}^{\text{out}} | 0^{\text{in}} \rangle =$$

$$= | \left(\beta_k a_{-\vec{k}}^{\text{in}\,\dagger} - i\sqrt{Z} \int d^4x e^{-i\vec{k}\cdot\vec{x}} (-\beta_k \Psi_k^{\text{in}} + \alpha_k \Psi_k^{\text{in}\,*} \right) | 0^{\text{in}} \rangle |^2 =$$

$$= \begin{cases} V|\beta_k|^2 \ + \ ... & (\beta_k \neq 0) \\ 0 \ + \ Z | \int d^4x e^{-i\vec{k}\cdot\vec{x}} \Psi_k^{\text{in}\,*} J | 0^{\text{in}} \rangle |^2 & (\beta_k = 0) \end{cases}$$

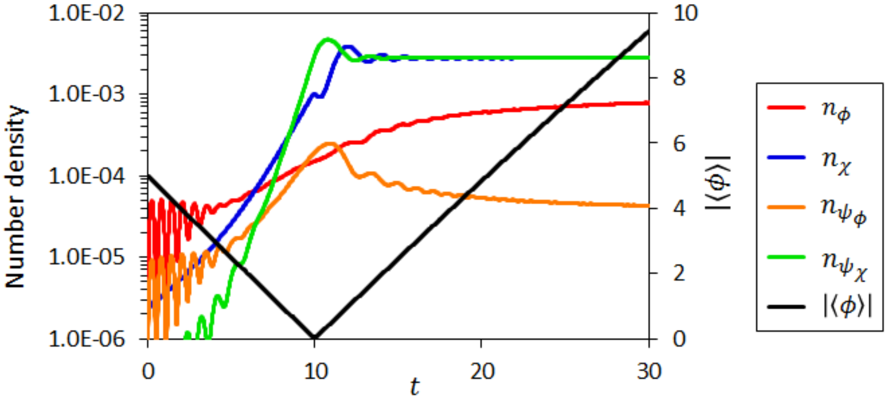
Particles are produced even if $\beta_k = 0$.

How big is that effect?

We consider the simple supersymmetric model with superpotential

$$W = \frac{g}{2} \Phi X^2.$$

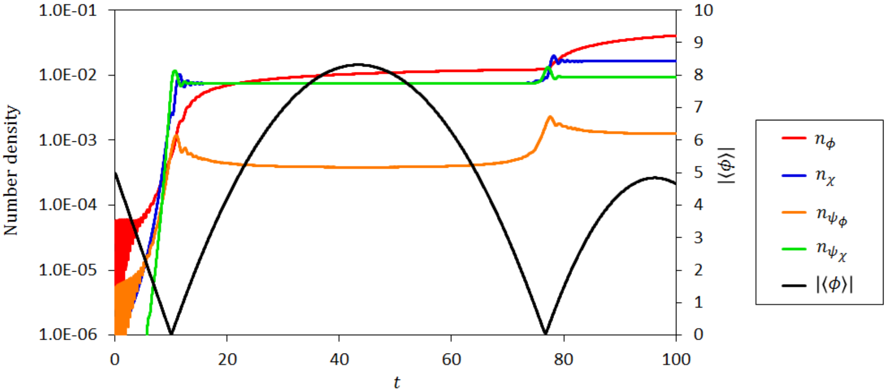
Results: one transition



$\phi(t=0) = 5 + 0.05i$, $\dot{\phi}(t=0) = -0.5$, $g = 1$

at $t = 30$: $n_\phi = 7.82 \cdot 10^{-4}$, $n_\chi = 2.77 \cdot 10^{-3}$, $n_{\psi_\phi} = 4.26 \cdot 10^{-5}$, $n_{\psi_\chi} = 2.78 \cdot 10^{-3}$

Results: trapping effect



$$\phi(t=0) = 5 + 0.05i,\ \dot{\phi}(t=0) = -0.5,\ g = 2$$

Expanding Universe

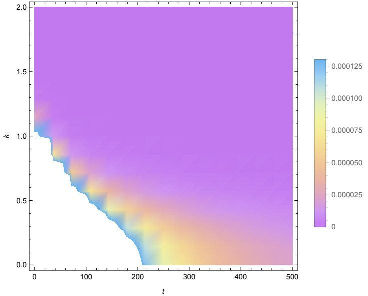
Simple model without interactions after one transition ($M_{PL} = 1$):

- $n^{\phi} = n^{\psi_\phi} \approx 0$
- $n^{\chi} = n^{\psi_\chi} = \frac{(gv)^{3/2}}{(2\pi)^3}\left(\frac{a_0}{a}\right)^3 e^{-\frac{\pi}{gv}\left(g^2\mu^2 + \frac{9w}{4}H_0^2\right)}$

where $H_0 \approx \frac{1}{\sqrt{3}}|v|$.

Cut-off momentum for massive fermions (additional shift):

$$k_{\max}^2/a_0^2 = \frac{gv}{\pi}\ln(2) - g^2\mu^2 - \frac{9w}{4}H_0^2.$$



With interactions for general scalar Ψ

$$n_k = \begin{cases} \frac{1}{a^3}V|\beta_k|^2 + \ldots & (\beta_k \neq 0) \\ 0 + \frac{1}{a^3}Z|\int d^4x e^{-i\vec{k}\cdot\vec{x}}\Psi_k^{\text{in}\,*} a^{\gamma} J_\Psi |0^{\text{in}}\rangle|^2 & (\beta_k = 0) \end{cases}$$

Conclusions

- general method of calculating particle production
  (in the suitable limit we recover parametric resonance)
- heavier particles are produced more efficiently
- number density of produed massless particles coming from rescattering is non negligible
- cut-off momentum in the distribution of produced particles is present

Back up slides

Examples of particle production

- positron-electron pair production (Schwinger effect) in oscillating electric field
- gravitational particle production due to changing metric
- SM particle production during preheating & reheating due to oscillating inflaton

Decomposition of Ψ^{as} (it's a free field: $J(x) = 0$)

$$\Psi^{as}(x) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \Big(\Psi_k^{as}(x^0) a_{\vec{k}}^{as} + \Psi_k^{as\,*}(x^0) a_{-\vec{k}}^{as\,\dagger} \Big)$$

and taking time-dependent inner product relation (comes from canonical commutation relations)

$$\dot{\Psi}_k^{as\,*}\Psi_k^{as} - \Psi_k^{as\,*}\dot{\Psi}_k^{as} = i/Z$$

$$a_{\vec{k}}^{as} = -iZ \int d^3x e^{-i\vec{k}\cdot\vec{x}} \Big(\dot{\Psi}_k^{as\,*}\Psi^{as} - \Psi_k^{as\,*}\dot{\Psi}^{as} \Big).$$

Quantization of the scalar field in curved spacetime

The simplest action for scalar field ϕ in FRW flat metric $\left(ds^2 = dt^2 - a^2(t) d^2\vec{x} \right)$:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{1}{2} m^2 \phi^2 \right)$$

Solving Euler-Lagrange equation gives the equation of motion for ϕ in conformal coordinates $(\eta, \vec{x})$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} - \frac{\nabla^2}{a^2}\phi + m^2\phi = 0$$

Reparametrization $\chi = a\phi$ gives the simple form of equation of motion (without the 1^{st} derivative)

$$\ddot{\chi} - \nabla^2\chi + \left(m^2 a^2 - \frac{\ddot{a}}{a} \right)\chi = 0$$

Quantization of the scalar field in curved spacetime

Second quantization: decomposition of the scalar field into modes $v_{\vec{k}}$

$$\chi(x,\eta) = \int \frac{d^3k}{(2\pi)^3}\left(e^{i\vec{k}\cdot\vec{x}} v^*_{\vec{k}}(\eta) a_{\vec{k}} + e^{-i\vec{k}\cdot\vec{x}} v_{\vec{k}}(\eta) a^\dagger_{\vec{k}}\right)$$

Solving equation of motion for the field χ means finding particular set of modes fulfilling:

$$\ddot{v}_{\vec{k}} + \omega^2_{\vec{k}}(\eta) v_{\vec{k}} = 0$$

which is the simple harmonic oscillator equation with time-dependent frequency:

$$\omega^2_{\vec{k}} = |\vec{k}|^2 + m^2_{\text{eff}}(\eta) = |\vec{k}|^2 + m^2 a^2 - \frac{\ddot{a}}{a}$$

Particles are produced due to the coupling between the scale factor and our field (energy is transferred).
Also: $v_k^{\text{in}} = \alpha_k v_k^{\text{out}} + \beta_k^* v_k^{\text{out}\,*}$.

Vacuum choice

In Minkowski space vacuum is the lowest energy-eigenstate of the Hamiltonian (one and only). All inertial observers will always measure the same vacuum.

In curved spacetime no set of mode functions is distinguished and you are free to choose whichever vacuum you like - Hamiltonian is explicitly time-dependent and there are no time-independent eigenvectors that can serve as vacuum.

We distinguish two types of vacuum in curved spacetime:

- instantenous vacuum
- adiabatic vacuum

Instantaneous vacuum

Hamiltonian in curved spacetime

$$H(\eta) = \frac{1}{2} \int d^3x \Big(\dot{\chi}^2 + (\nabla\chi)^2 + m_{\text{eff}}^2 \chi^2 \Big)$$

and instantaneous vacuum at η_0 is just the lowest energy-state of the instantaneous Hamiltonian $\hat{H}(\eta_0)$. We compute the expectation value $\langle 0_v | \hat{H}(\eta_0) | 0_v \rangle$ and minimize it with respect to some chosen modes $v_k(\eta)$.
Vacuum expectation value of Hamiltonian is then

$$\langle 0_v | \hat{H}(\eta_0) | 0_v \rangle = \frac{1}{4} \delta^3(0) \int d^3k \Big(|v_k'|^2 + \omega_k^2(\eta) |v_k|^2 \Big)$$

and provided the normalization condition: $\text{Im}\big(v' v^*\big) = 1$, we find the initial conditions that determine the mode functions defining instantaneous vacuum

$$v_k(\eta_0) = \frac{1}{\sqrt{\omega_k(\eta_0)}} e^{i\gamma_k(\eta_0)} \qquad \& \qquad v_k'(\eta_0) = i\omega_k(\eta_0) v_k(\eta_0)$$

with arbitrary phase γ_k, asumming that $\omega_k^2 > 0$.

Adiabatic vacuum

The procedure is based on the WKB approximation for the solution of

$$\ddot{\chi}_k(\eta) + \omega^2(\eta)\chi_k(\eta) = 0$$

with slowly changing background. Our ansatz for the vacuum

$$\chi_k(\eta) = \frac{1}{\sqrt{W_k(\eta)}} e^{i\int\limits_{\eta_0}^{\eta} W_k(\eta) d\eta}$$

determines the equation for W_k

$$W_k^2 = \omega_k^2 - \frac{1}{2}\left[\frac{W_k''}{W_k} - \frac{3}{2}\left(\frac{W_k'}{W_k}\right)^2\right]$$

For a slowly changing spacetime: ${W_k'}^2/W_k^2, W_k''/W_k \ll \omega_k^2$, so as a 0^{th} approximation

$$W_k^{(0)}(\eta) = \omega_k(\eta)$$

Higher orders can be estimated by iteration.
For 0^{th} order:

$$\frac{1}{4} E_k^{\text{adiabatic}} = \frac{1}{4} E_k^{\text{instantaneous}} + \frac{\omega_k}{16}\epsilon^2, \text{ where } \epsilon = \frac{\dot{\omega}_k}{\omega_k^2}$$

SUSY-breaking

We can include SUSY-breaking in our model in two ways:

- SUSY can be broken from the beginning, before the production occurs
- SUSY can be broken after the production and before rescattering

In both cases SUSY is broken during scattering and decays of produced particles after production.

We consider two possible soft SUSY-breaking terms:

- $\delta\mathcal{L}_{soft} = m^2|\chi|^2$
- $\delta\mathcal{L}_{soft} = m^2|\chi|^2 + A\phi\chi^2 + h.c.$

Second possibility is crucial in gravity-mediation scenarios of SUSY-breaking, when $A \sim m$, in other scenarios it just come down to the first one ($A \ll m$).

M2: Number density of produced particles, scalars

Equations of motion for scalars:

$$\partial^2\chi + (g^2 + h^2)|\phi|^2\chi + gh|\phi|^2\psi = 0$$
$$\partial^2\psi + gh|\phi|^2\chi + h^2|\phi|^2\psi = 0$$

fields χ and ψ are mixed.

After diagonalization of mass matrix, e.o.m. for mass eigenstates:

$$\partial^2\chi'_k + \left(m^2_{\chi'} + k^2\right)\chi'_k = 0$$
$$\partial^2\psi'_k + \left(m^2_{\psi'} + k^2\right)\psi'_k = 0$$

where

$$m^2_{\chi'} = |\phi|^2\left(\frac{g^2+2h^2}{2} + \frac{g}{2}\sqrt{g^2 + 4h^2}\right) = |\phi|^2\tilde{g}^2$$
$$m^2_{\psi'} = |\phi|^2\left(\frac{g^2+2h^2}{2} - \frac{g}{2}\sqrt{g^2 + 4h^2}\right) = |\phi|^2\tilde{h}^2$$

Mass eigenstates are free: there are no interaction terms coming from the derivative of diagonalizing matrix which is constant.

M2: Number density of produced particles

Number density of produced bosons:

- $n_{\chi'} = 2\frac{(\tilde{g}v_\phi)^{3/2}}{(2\pi)^3} e^{-\pi\tilde{g}\frac{\mu_\phi^2}{v_\phi}}$
- $n_{\psi'} = 2\frac{(\tilde{h}v_\phi)^{3/2}}{(2\pi)^3} e^{-\pi\tilde{h}\frac{\mu_\phi^2}{v_\phi}}$

Masses of fermionic mass-eigenstates:

- $m_{\tilde{\chi}'} = \frac{1}{2}\langle\phi\rangle\left(g + \sqrt{g^2 + 4h^2}\right)$
- $m_{\tilde{\psi}'} = \frac{1}{2}\langle\phi\rangle\left(g - \sqrt{g^2 + 4h^2}\right)$

Number density of produced fermions:

- $n_{\tilde{\chi}'} = 2\frac{(g'v)^{3/2}}{(2\pi)^3} e^{-\pi g'\mu^2/v}$
- $n_{\tilde{\psi}'} = 2\frac{(h'v)^{3/2}}{(2\pi)^3} e^{-\pi h'\mu^2/v}$

where

$$g' = \frac{1}{2}\left(g + \sqrt{g^2 + 4h^2}\right)$$
$$h' = \frac{1}{2}|g - \sqrt{g^2 + 4h^2}|$$

Calculation

Plan of calculation

- equations of motions with proper source terms for our fields: ϕ, χ, ψ_ϕ, ψ_χ
- Yang-Feldman equations and definition of asymptotic fields
- inner product relations (different for bosons and fermions)
- relation between "in" and "out" fields (Bogoliubov transformation)
- identifying β_k coefficients what leads to number density estimation

Simple for massive particles. Massless particles need WKB approximation and numerical analysis.

List of the things to do

- interactions of 3 fields
- comparing with parametric resonance and thermal production
- higher order calculations
- scale factor
- applications
- ...